Chapter 5

5 - 10

5. $r = 10$

$$s = r\theta$$
$$= 10\left(\frac{2\pi}{5}\right)$$
$$= 4\pi$$

6. $s = r\theta$

$$7 = 2.5\theta$$
$$\theta = 2.8 \text{ rad}$$
$$= \frac{2.8}{\pi} \times 180^\circ$$
$$= 160.4^\circ$$

11 - 14

11. Area of a circle

$= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} (5)^2 (2)$

$= 25$

15. $r = 8$

$s = r\theta$

Let $\frac{\theta}{t} = w$,

$$v = \frac{s}{t}$$

$$= \frac{r\theta}{t}$$

$$= rw$$

$$v = wr$$

Linear speed

$= v$

$= 150 \text{ rpm}$

$= 150(2\pi r) \text{pm}$

$= \frac{300\pi \text{ inch}}{1 \text{ min}}$

Angular speed

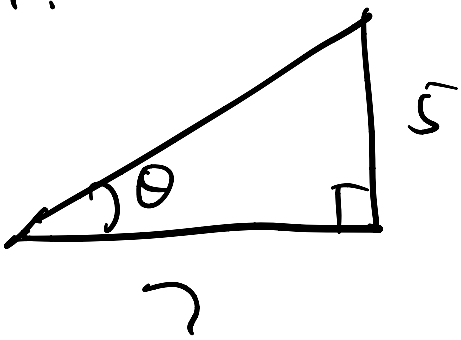
$= w$

$= \frac{v}{r}$

$= \frac{300\pi}{8}$

$= 37.5\pi$ rad / minute

17.



$$h = \sqrt{5^2 + 7^2}$$
$$= \sqrt{74}$$

$$\sin\theta = \frac{5}{\sqrt{74}} \qquad \cot\theta = \frac{7}{5}$$

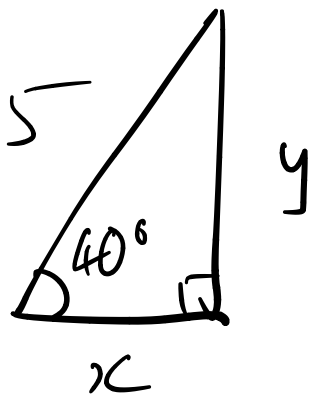
$$\cos\theta = \frac{7}{\sqrt{74}}$$

$$\tan\theta = \frac{5}{7}$$

$$\csc\theta = \frac{\sqrt{74}}{5}$$

$$\sec\theta = \frac{\sqrt{74}}{7}$$

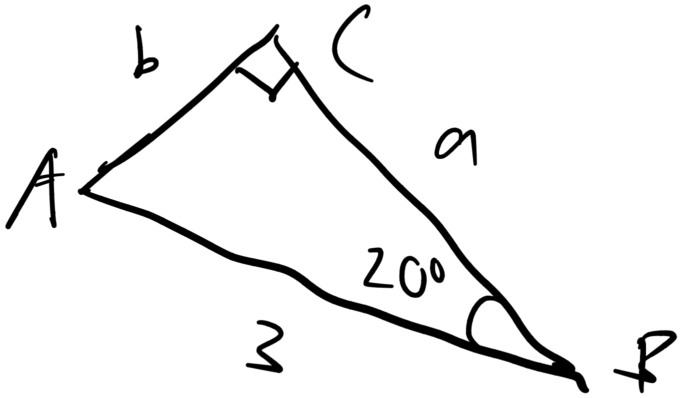
19.



$$y = 5\sin 40^\circ$$
$$= 3.21$$

$$x = 5\cos 40^\circ$$
$$= 3.83$$

23.



$$\angle A = 180^\circ - 90^\circ - 20^\circ$$
$$= 70^\circ$$

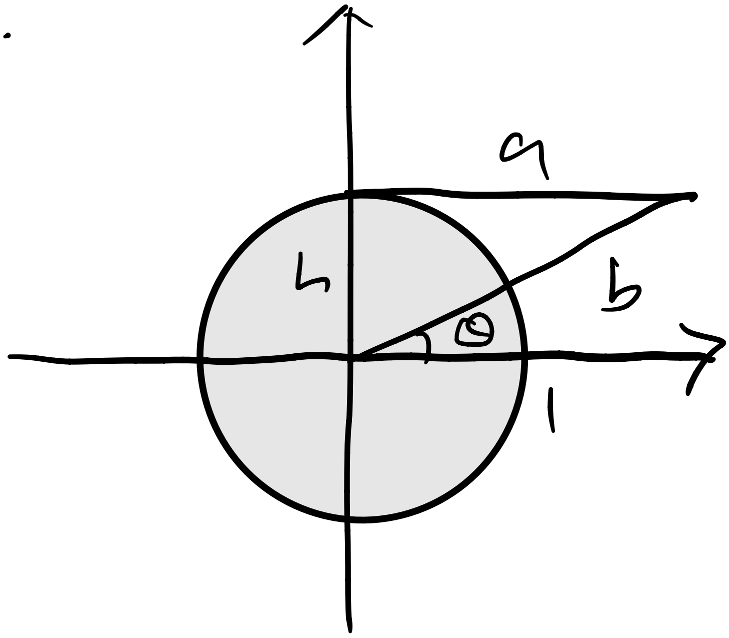
Sine Rule

$$\frac{\sin C}{3} = \frac{\sin B}{b} \qquad \frac{\sin A}{a} = \frac{\sin C}{c}$$

$$a = \frac{\sin A}{\sin C} \cdot c$$
$$= \frac{\sin 70^\circ}{\sin 90^\circ} \cdot 3$$
$$= 2.82$$

$$b = \frac{\sin B}{\sin C} \cdot 3$$
$$= \frac{3 \sin 20}{\sin 90^\circ}$$
$$= 1.03$$

27.



$h = 1$

$$\tan\theta = \frac{h}{a}$$

$$a = \frac{1}{\tan\theta} = \cot\theta$$

$$\sin\theta = \frac{h}{b}$$

$$b = \frac{1}{\sin\theta} = \csc\theta$$

33. $\sin 315^\circ$

$$\text{ref } \theta = 360^\circ - 315^\circ$$
$$= 45^\circ$$

$$\theta = -\sin 45^\circ$$
$$= -\frac{\sqrt{2}}{2}$$

47. $y-\sqrt{3}x+1=0$

$y=0$

x-intercept:

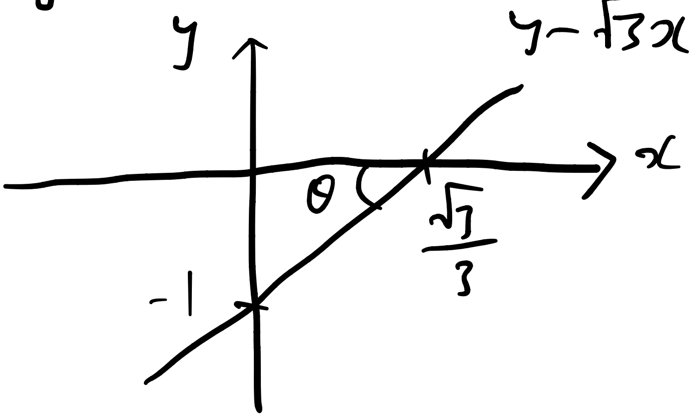
$0-\sqrt{3}x+1=0$

$-\sqrt{3}x=-1$

$x=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$

y-intercept:

$y=-1$



$\theta=\tan^{-1}-\sqrt{3}$

$=120°, -60°$

$\therefore \theta=60°$, acute angle

$\tan\theta=\frac{-1}{\frac{\sqrt{3}}{3}}$

$=-\frac{3}{\sqrt{3}}$

$=-\frac{3\sqrt{3}}{3}=-\sqrt{3}$

49. $\tan\theta$, $\cos\theta$, θ in Quadrant II

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\pm\sqrt{1-\cos^2\theta}}{\cos\theta}$$

$$= \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$$

$\sin\theta$ in Q II

53. $\tan\theta = \frac{\sqrt{7}}{3}$, $\sec\theta = \frac{4}{3}$

$\cot\theta = \frac{3}{\sqrt{7}}$, $\cos\theta = \frac{3}{4}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin\theta = \tan\theta\cos\theta$$

$$= \frac{\sqrt{7}}{\cancel{3}} \cdot \frac{\cancel{3}}{4}$$

$$= \frac{\sqrt{7}}{4}$$

$\csc\theta = \frac{4}{\sqrt{7}}$

57. $\tan\theta = -\frac{1}{2}$, Quadrant II

$$\sin\theta = \frac{1}{\sqrt{5}}, \cos\theta = -\frac{2}{\sqrt{5}}$$

$$\sin\theta + \cos\theta = \frac{1}{\sqrt{5}} - \frac{2}{\sqrt{5}}$$

$$= \frac{\sqrt{5} - 2\sqrt{5}}{5}$$

$$= \frac{-\sqrt{5}}{5}$$

61. $\sin^{-1}(\sqrt{3}/2)$

$$= \frac{\pi}{3}\text{rad} \;/\; 60^\circ$$

65. $\sin(\tan^{-1} x)$

$1 + \cot^2\theta = \sec^2\theta$

$1 + \frac{1}{\tan^2\theta} = \frac{1}{\cos^2\theta}$

$\cos^2\theta + \frac{\cos^2\theta}{\tan^2\theta} = 1$

$\cos^2\theta\tan^2\theta + \cos^2\theta = \tan^2\theta$

$\cos^2\theta = \frac{\tan^2\theta}{\tan^2\theta + 1}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \frac{\tan^2\theta}{\tan^2\theta + 1}$

$= \frac{\tan^2\theta + 1 - \tan^2\theta}{\tan^2\theta + 1}$

$= \frac{1}{\tan^2\theta + 1}$

$\sin\theta = \pm\frac{1}{\sqrt{\tan^2\theta + 1}}$

Let $u = \tan^{-1} x$

$0 \leq u \leq \pi$,

as $\tan^{-1} x$ is only defined within this range,

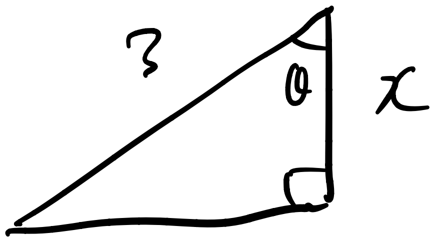
$\therefore \sin u$ is positive

$\sin(\tan^{-1} x)$

$= \pm\frac{1}{\sqrt{\tan^2(\tan^{-1} x) + 1}}$

$= \frac{1}{\sqrt{x^2 + 1}}$

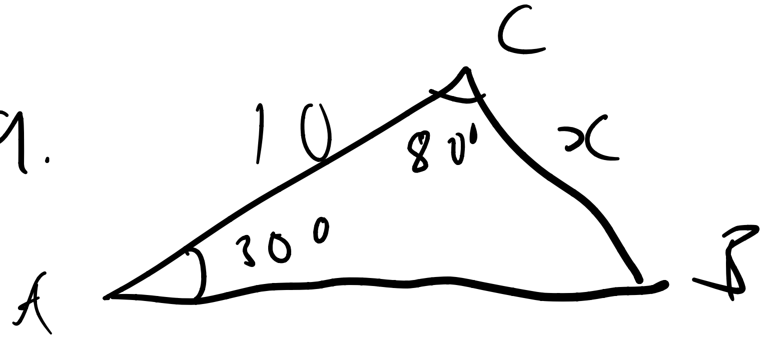
67.



$$3\cos\theta = x$$

$$\theta = \cos^{-1}\left(\frac{x}{3}\right)$$

69.

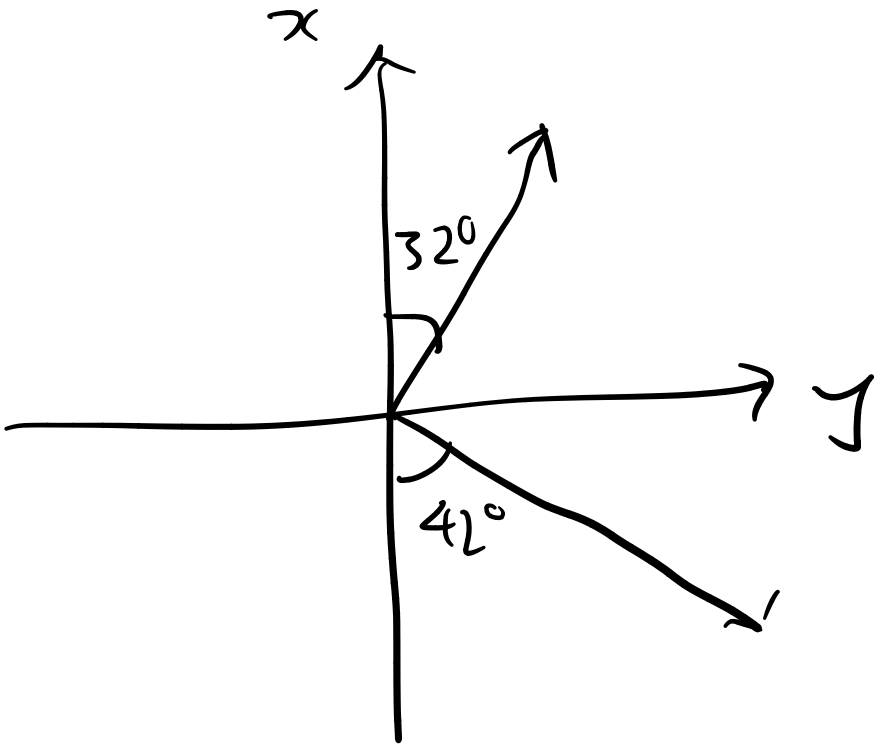


$$\angle B = 180 - 80 - 30$$
$$= 70^\circ$$

$$\frac{\sin B}{b} = \frac{\sin A}{x}$$

$$x = \frac{\sin A}{\sin B} \cdot b$$
$$= \frac{10 \sin 30^\circ}{\sin 70^\circ}$$
$$= 5.32$$

79.



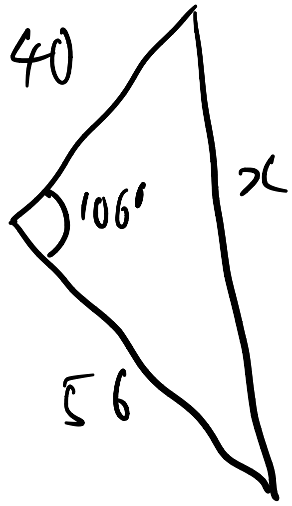
After 2 hrs,

Ship at N 32° E,

$d_1 = 20 \times 2 = 40$ mi

Ship at S 42° E,

$d_2 = 28 \times 2 = 56$ mi



$$x^2 = a^2 + b^2 - 2ab\cos x$$
$$= 40^2 + 56^2 - 2(40)(56)\cos 106^\circ$$
$$= 5970.86$$
$$x = 77.3 \text{ mi}$$

Chapter 6

6.1 The Unit Circle

a. Terminal Points

b. Reference Numbers

c. Terminal Points & Reference Numbers

6.1

Terminal Points, 23–36

3. $x^2+y^2=r^2$

if unit circle, $r=1$

$$x^2+y^2=\left(\frac{3}{5}\right)^2+\left(-\frac{4}{5}\right)^2$$

$$=\frac{9}{25}+\frac{16}{25}$$

$$=1$$

$\therefore$ point is on the unit circle

9. $x=-\frac{3}{5}$

$$x^2+y^2=1^2$$

$$\left(-\frac{3}{5}\right)^2+y^2=1$$

$$y^2=1-\frac{9}{25}$$

$$y^2=\frac{16}{25}$$

$y=-\frac{4}{5}$ Quadrant $\underline{III}$

23.

24. $t = -3\pi$

$t = \pi$

$\therefore P(-1, 0)$

25. $t = \frac{3\pi}{2}$

$\therefore P(0, -1)$

26. $t = \frac{5\pi}{2}$

$t = \frac{\pi}{2}$

$\therefore P(0, 1)$

27. $t = -\frac{\pi}{6}$

$\therefore P\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

28. $t = \frac{7\pi}{6}$

$t = \frac{\pi}{6} + \pi$

$\therefore P\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

29. $t = \frac{5\pi}{4}$

$t = \frac{\pi}{4} + \pi$

$\therefore P\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

30. $t = \frac{4\pi}{3}$

$t = \frac{\pi}{3} + \pi$

$\therefore P\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

31. $t = -\frac{7\pi}{6}$

$t = -\frac{\pi}{6} - \pi$

$t = \frac{5\pi}{6}$

$\therefore P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

32. $t = \frac{5\pi}{3}$

$t = \frac{2\pi}{3} + \pi$

$\therefore P\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

33. $t = -\frac{7\pi}{4}$

$t = -\frac{3\pi}{4} - \pi$

$\therefore P\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

34. $t = -\frac{4\pi}{3}$

$t = -\frac{\pi}{3} - \pi$

$\therefore P\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

35. $t = -\frac{3\pi}{4}$

$\therefore P\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

36. $t = \frac{11\pi}{6}$

$t = \frac{5\pi}{6} + \pi$

$P\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

23. $t = 4\pi$

$= 0\pi$

$\therefore P(1, 0)$

27. $t = -\dfrac{\pi}{6}$

$r = 1$

$P(x, y)$

$= P(\cos t, \sin t)$

$x = \cos t$

$= \cos\left(-\dfrac{\pi}{6}\right)$

$= \cos\left(\dfrac{\pi}{6}\right)$

$= \dfrac{\sqrt{3}}{2}$

$y = \sin t$

$= \sin\left(-\dfrac{\pi}{6}\right)$

$= -\sin\left(\dfrac{\pi}{6}\right)$

$= -\dfrac{1}{2}$

$\cos\dfrac{\pi}{6} = \dfrac{x}{h}$

$= \dfrac{x}{1}$

$\cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$

$x = \dfrac{\sqrt{3}}{2}$

37. (a) $t = \frac{4\pi}{3}$ (b) $t = \frac{5\pi}{3}$

$\bar{t} = 2\pi - \frac{5\pi}{3}$

$\bar{t} = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$ $= \frac{\pi}{3}$

(c) $t = -\frac{7\pi}{6}$

$\bar{t} = -\pi - \left(-\frac{7\pi}{6}\right)$

$= \frac{\pi}{6}$

(d) $t = 3.5$

$\bar{t} = 3.5 - \pi$

$= 0.358$

38. (a) $t = 9\pi$

$t = 9\pi - 8\pi$

$= \pi$

$\bar{t} = \pi - \pi$

$= 0$

(b) $t = -\frac{5\pi}{4}$

$\bar{t} = -\pi - \left(-\frac{5\pi}{4}\right)$

$= \frac{\pi}{4}$

(c) $t = \frac{25\pi}{6}$

$t = \frac{25\pi}{6} - \frac{24\pi}{6}$

$= \frac{\pi}{6}$

$\bar{t} = \frac{\pi}{6}$

(d) $t = 4$

$\bar{t} = 4 - \pi$

$= 0.858$

39. (a) $t = \frac{5\pi}{7}$

$\bar{t} = \pi - \frac{5\pi}{7}$

$= \frac{2\pi}{7}$

(b) $t = -\frac{7\pi}{9}$

$\bar{t} = -\frac{7\pi}{9} - (-\pi)$

$= \frac{2\pi}{9}$

(c) $t = -3$

$\bar{t} = -3 - (-\pi)$

$= 0.142$

(d) $t = 5$

$\bar{t} = 2\pi - 5$

$= 1.283$

40. (a) $t = \frac{11\pi}{5}$

$t = \frac{11\pi}{5} - \frac{10\pi}{5}$

$= \frac{\pi}{5}$

$\bar{t} = \frac{\pi}{5}$

(b) $t = -\frac{9\pi}{7}$

$\bar{t} = -\pi - \left(-\frac{9\pi}{7}\right)$

$= \frac{2\pi}{7}$

(c) $t = 6$

$\bar{t} = 2\pi - 6$

$= 0.283$

(d) $t = -7$

$\bar{t} = -2\pi - (-7)$

$= 0.717$

41. $t = \frac{11\pi}{6}$

(a) $\bar{t} = 2\pi - \frac{11\pi}{6}$

$= \frac{\pi}{6}$

(b) $\bar{t}$, $Q(\cos\bar{t}, \sin\bar{t})$

$= Q\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

t, $P\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

47. $t = \frac{13\pi}{4} - 2\pi = \frac{5\pi}{4}$

(a) $\bar{t} = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$

(b) $\bar{t}$, $Q\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

t, $P\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

61. $\overset{\curvearrowright}{PQ} = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$

$\overset{\curvearrowright}{PR} = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

$\overline{PQ} = \overline{PR}$

$$d(P,Q) = \sqrt{(x-x)^2 + (y-(-y))^2}$$
$$= \sqrt{4y^2}$$
$$= 2y$$

$$d(P,R) = \sqrt{(x-0)^2 + (y-1)^2}$$
$$= \sqrt{x^2 + y^2 - 2y + 1}$$

$$2y = \sqrt{x^2 + y^2 - 2y + 1}$$
$$4y^2 = x^2 + y^2 - 2y + 1$$
$$4y^2 + 2y - 2 = 0$$
$$2y^2 + y - 1 = 0$$

$$(2y-1)(y+1) = 0$$
$$y = \frac{1}{2}, -1$$
$$y \neq -1,$$
$$\therefore y = \frac{1}{2}$$

$$x^2 + y^2 = 1$$
$$x^2 + \left(\frac{1}{2}\right)^2 = 1$$
$$x^2 = \frac{3}{4}$$
$$x = \pm\frac{\sqrt{3}}{2}$$
$$\therefore x = \frac{\sqrt{3}}{2}$$

$$\therefore P\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

6.2 Trigonometric Functions of Real Numbers

3. $\sin t = n\frac{\pi}{4}$, $n = [1, 8]$

$n=1$, $\sin t = \frac{\sqrt{2}}{2}$, $\cos t = \frac{\sqrt{2}}{2}$

$n=2$, $\sin\frac{\pi}{2} = 1$, $\cos\frac{\pi}{2} = 0$

$n=3$, $\sin\frac{3\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

$n=4$, $\sin\pi = \sin 0 = 0$, $\cos\pi = -1$

$n=5$, $\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$, $\cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

$n=6$, $\sin\frac{3\pi}{2} = -1$, $\cos\frac{3\pi}{2} = 0$

$\bar{t} = \frac{\pi}{2}$

$n=7$, $\sin\frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$, $\cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$

$\bar{t} = \frac{\pi}{4}$

$n=8$, $\sin 2\pi = \sin 0 = 0$, $\cos 2\pi = \cos 0 = 1$

5. (a) $\sin \frac{7\pi}{6}$

$$\bar{t} = \frac{7\pi}{6} - \pi = \frac{\pi}{6}$$

$$\sin \frac{7\pi}{6} = -\sin \frac{\pi}{6}$$

$$= -\frac{1}{2}$$

(b) $\cos \frac{17\pi}{6} = \cos \frac{5\pi}{6}$

$$\bar{t} = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$$

$$\cos \frac{17\pi}{6} = -\cos \frac{\pi}{6}$$

$$= -\frac{\sqrt{3}}{2}$$

(c) $\tan \frac{7\pi}{6}$

$$\bar{t} = \frac{7\pi}{6} - \pi = \frac{\pi}{6}$$

$$\tan t = \tan \bar{t}$$

$$= \frac{\sqrt{3}}{3}$$

Evaluating Trigonometric Functions

6. (a) $\sin\frac{5\pi}{3}$

$t = \sin\frac{2\pi}{3}$

$\bar{t} = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

$\sin t = \sin\bar{t}$

$= \sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

(b) $\cos\frac{11\pi}{3}$

$= \cos\frac{5\pi}{3}$

$\bar{t} = 2\pi - \frac{5\pi}{3}$

$= \frac{\pi}{3}$

$\cos t = \cos\bar{t}$

$= \cos\frac{\pi}{3}$

$= \frac{1}{2}$

(c) $\tan\frac{5\pi}{3}$

$\bar{t} = 2\pi - \frac{5\pi}{3}$

$= \frac{\pi}{3}$

$\tan t = -\tan\bar{t}$

$= -\tan\frac{\pi}{3}$

$= -\sqrt{3}$

13. (a) $\cos\left(-\frac{\pi}{3}\right)$

$= \cos\frac{\pi}{3}$

$= \frac{1}{2}$

(b) $\sec\left(-\frac{\pi}{3}\right)$

$= \sec\left(\frac{\pi}{3}\right)$

$= 2$

(c) $\sin\left(-\frac{\pi}{3}\right) = -\sin\frac{\pi}{3}$

$= -\frac{\sqrt{3}}{2}$

39. $\sin 1.2$

(a) $\sin 1.2 = 0.75$

(b) $\sin 1.2 = 0.93$

41. $\tan 0.8$

(a) 1.00

(b) 1.03

53. $\sin t$, $\cos t$; Quadrant II

$$\sin t = \pm\sqrt{1-\cos^2 t}$$
$$= \sqrt{1-\cos^2 t}$$

sint is + in Q II

63. $\sin t = -\dfrac{4}{5}$

$r = 5$

$\csc t = -\dfrac{5}{4}$

$$x = \sqrt{5^2 - 4^2}$$
$$= \sqrt{9}$$
$$= 3$$

$\tan t = -\dfrac{4}{3}$

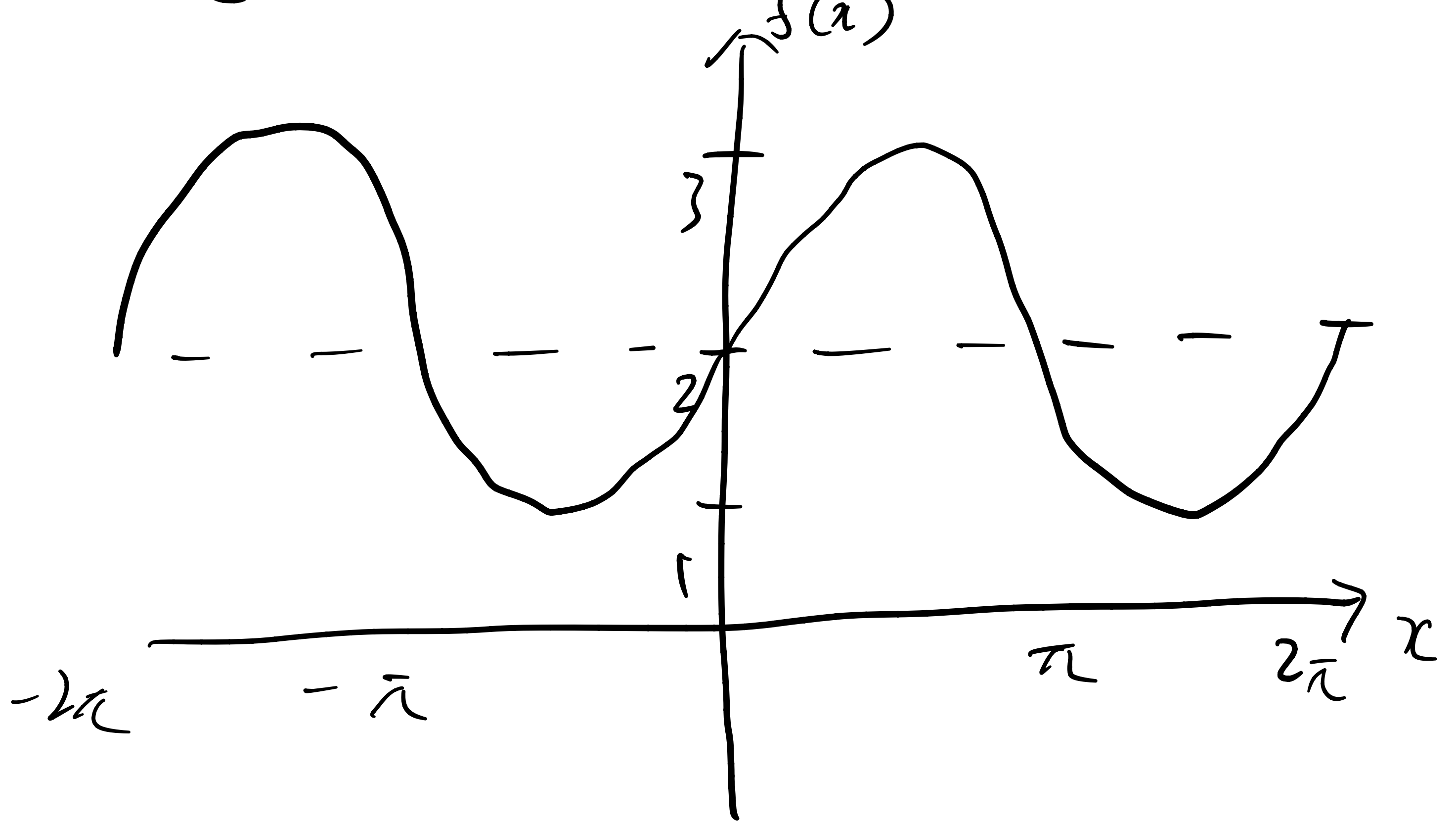
$\cot t = -\dfrac{3}{4}$

$\cos t = \dfrac{3}{5}$

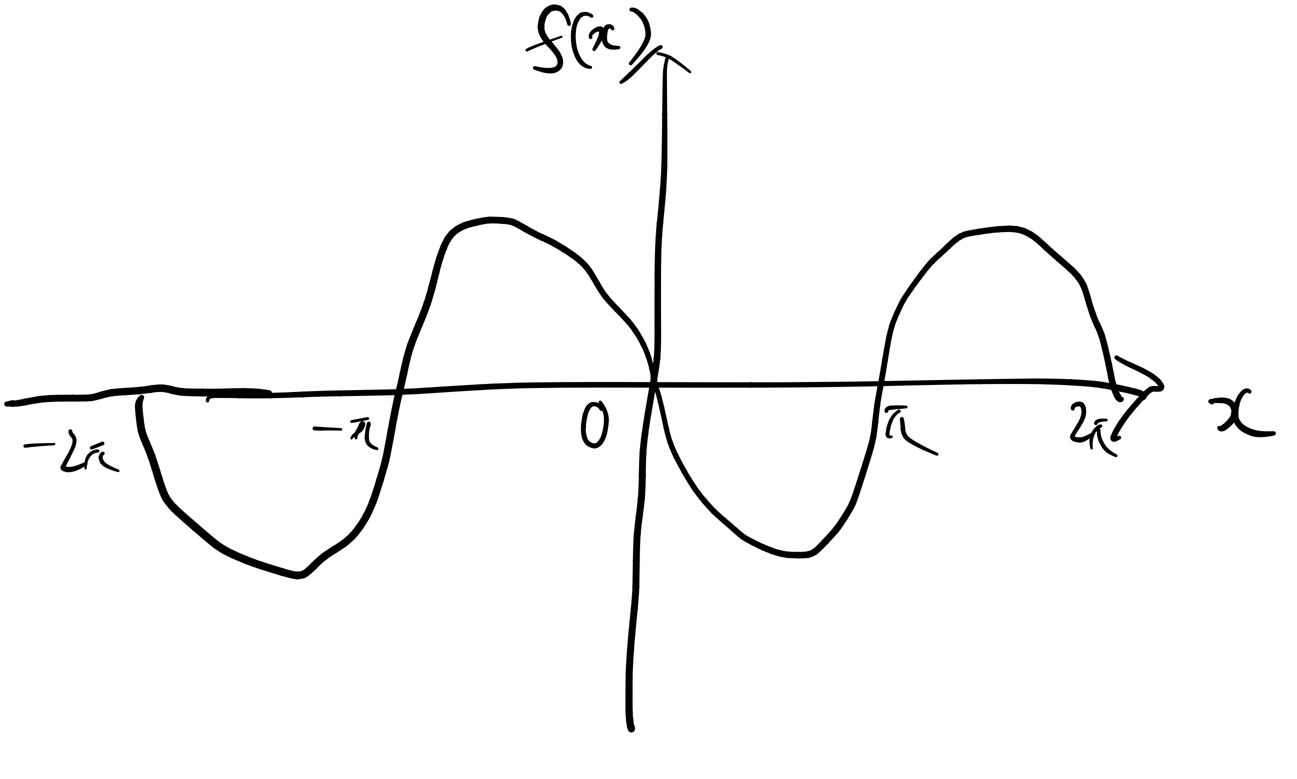
$\sec t = \dfrac{5}{3}$

6.3

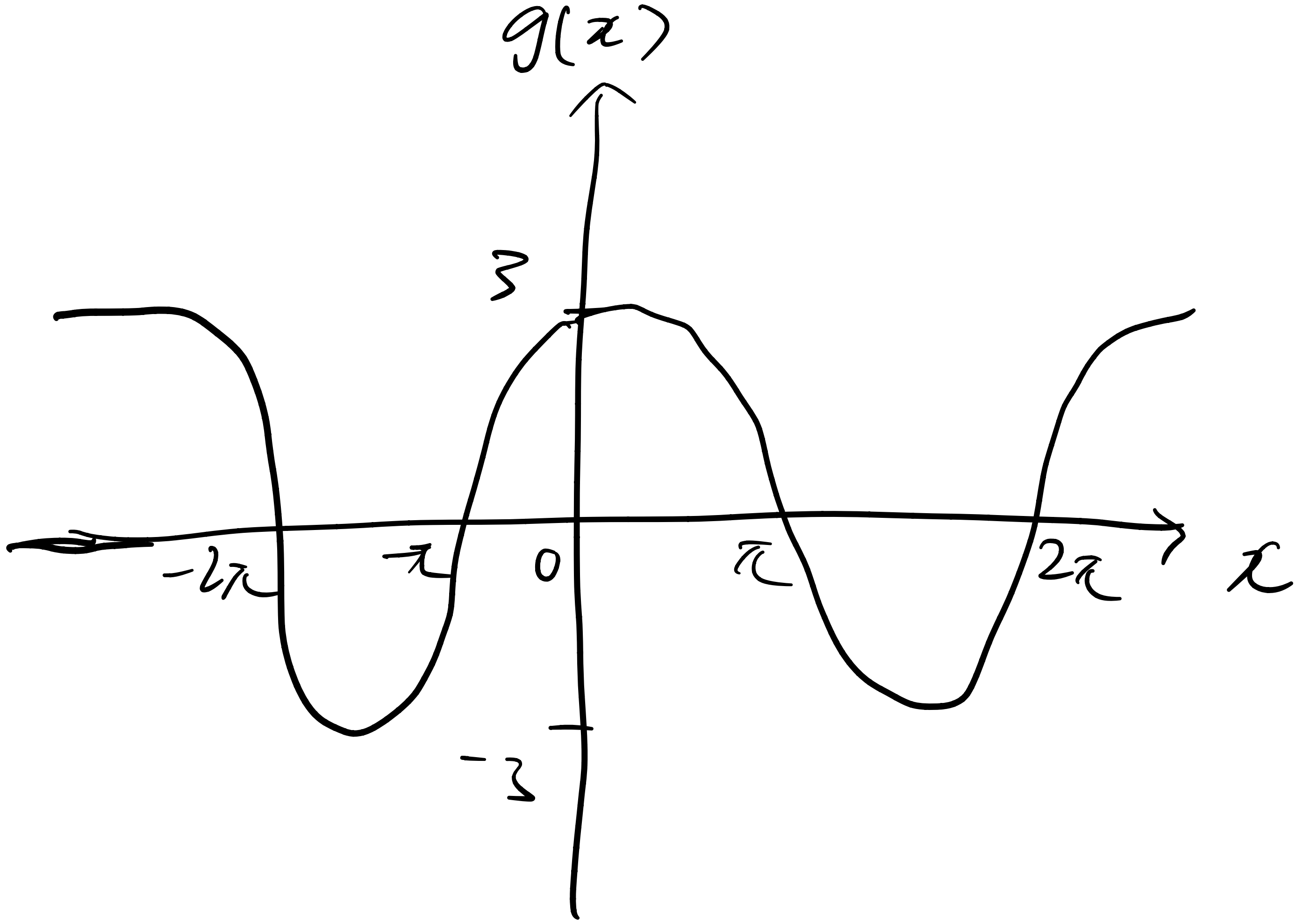
5. $f(x) = 2 + \sin x$



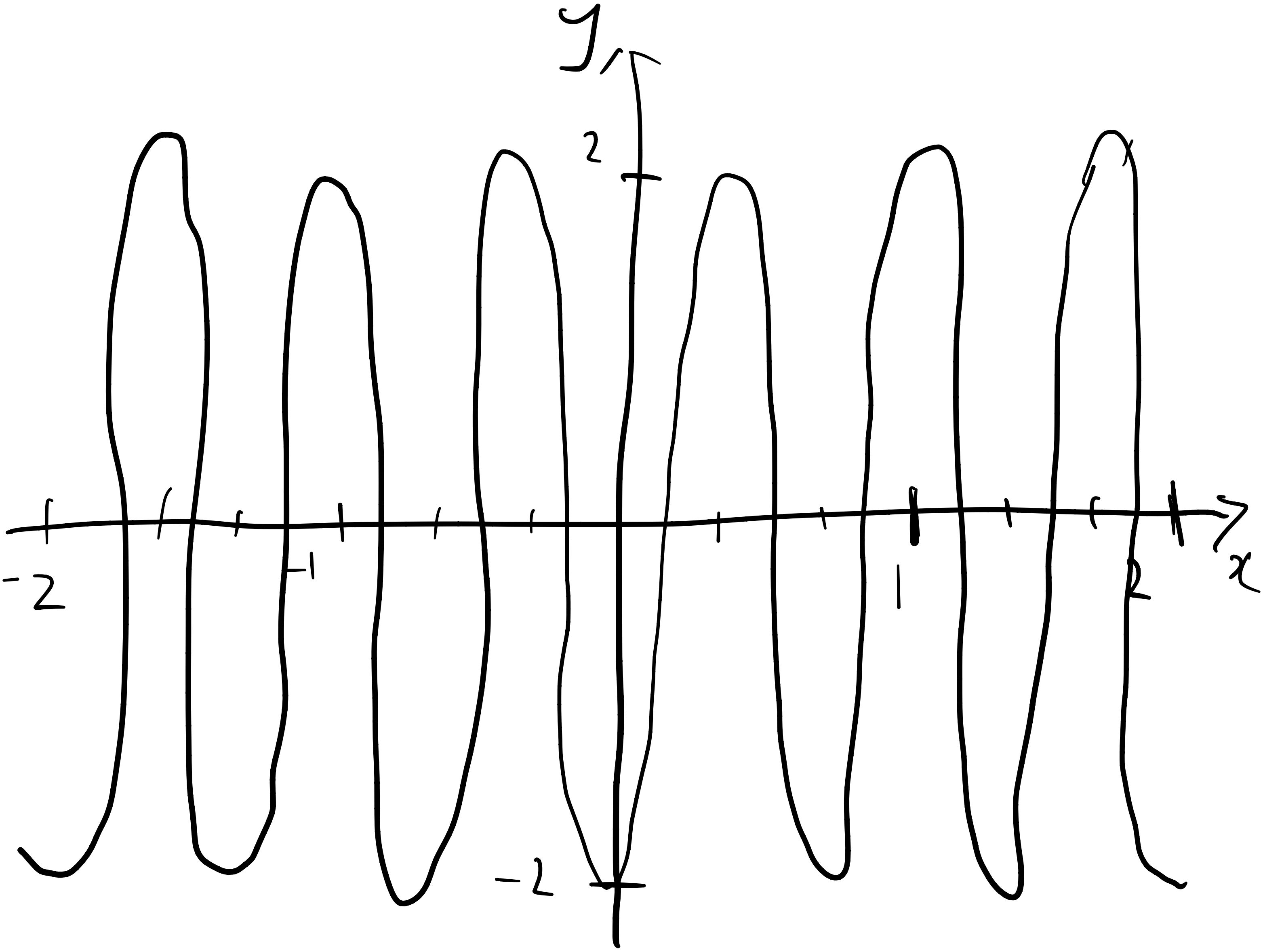
7. $f(x) = -\sin x$



11. $g(x) = 3\cos x$

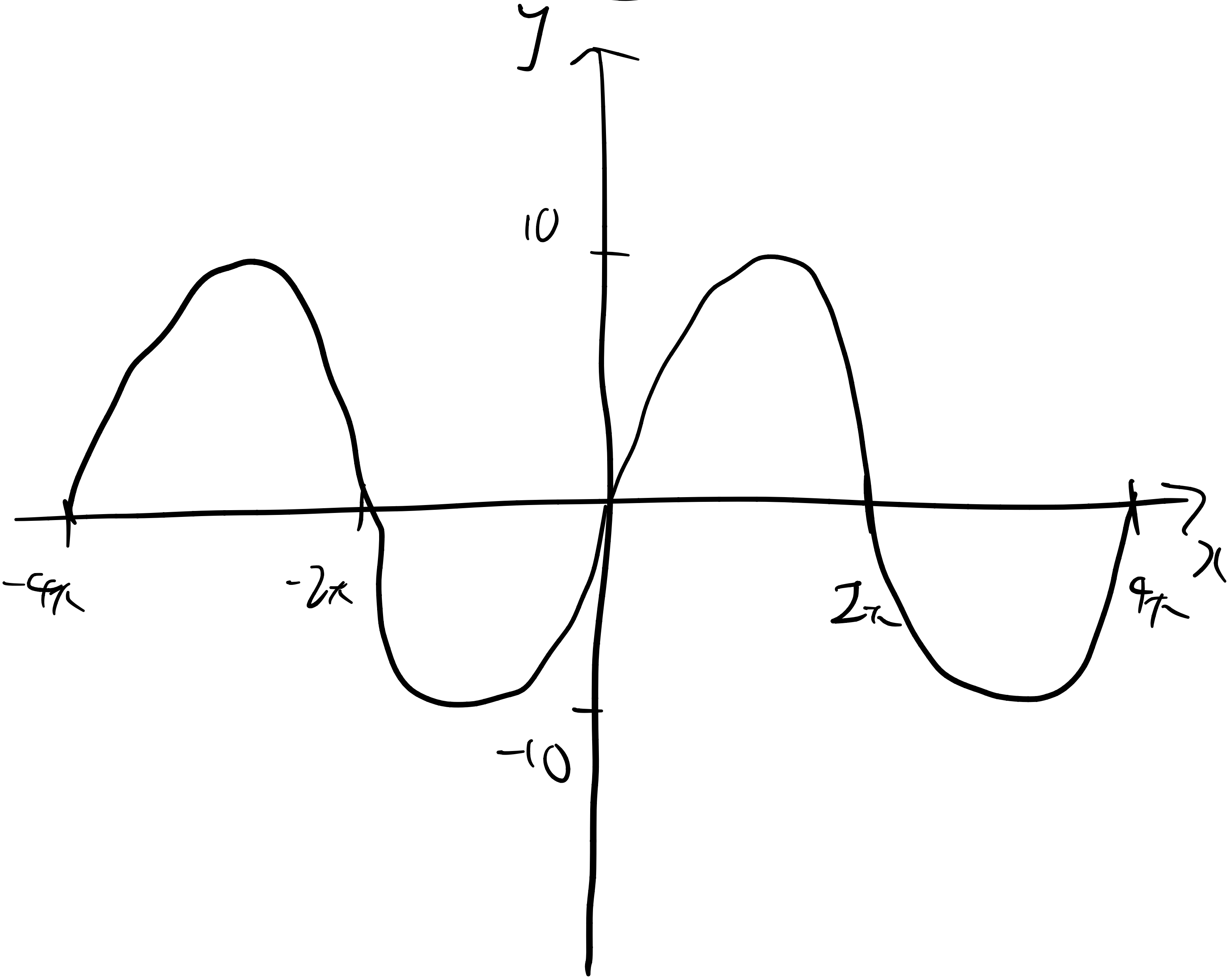


23. $y = -2\cos 3\pi x$

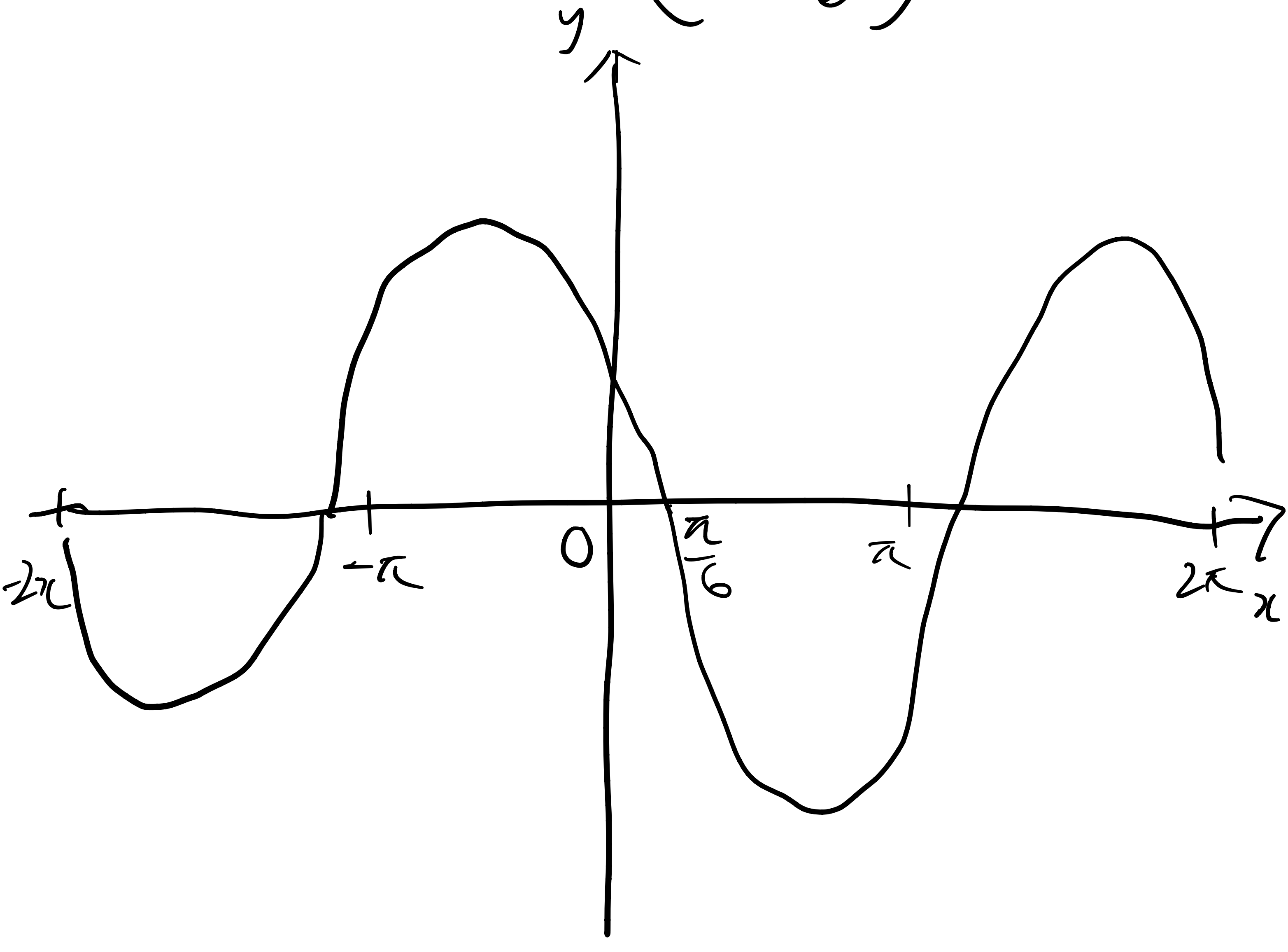


amplitude $= 2$, period $= \frac{2}{3}$

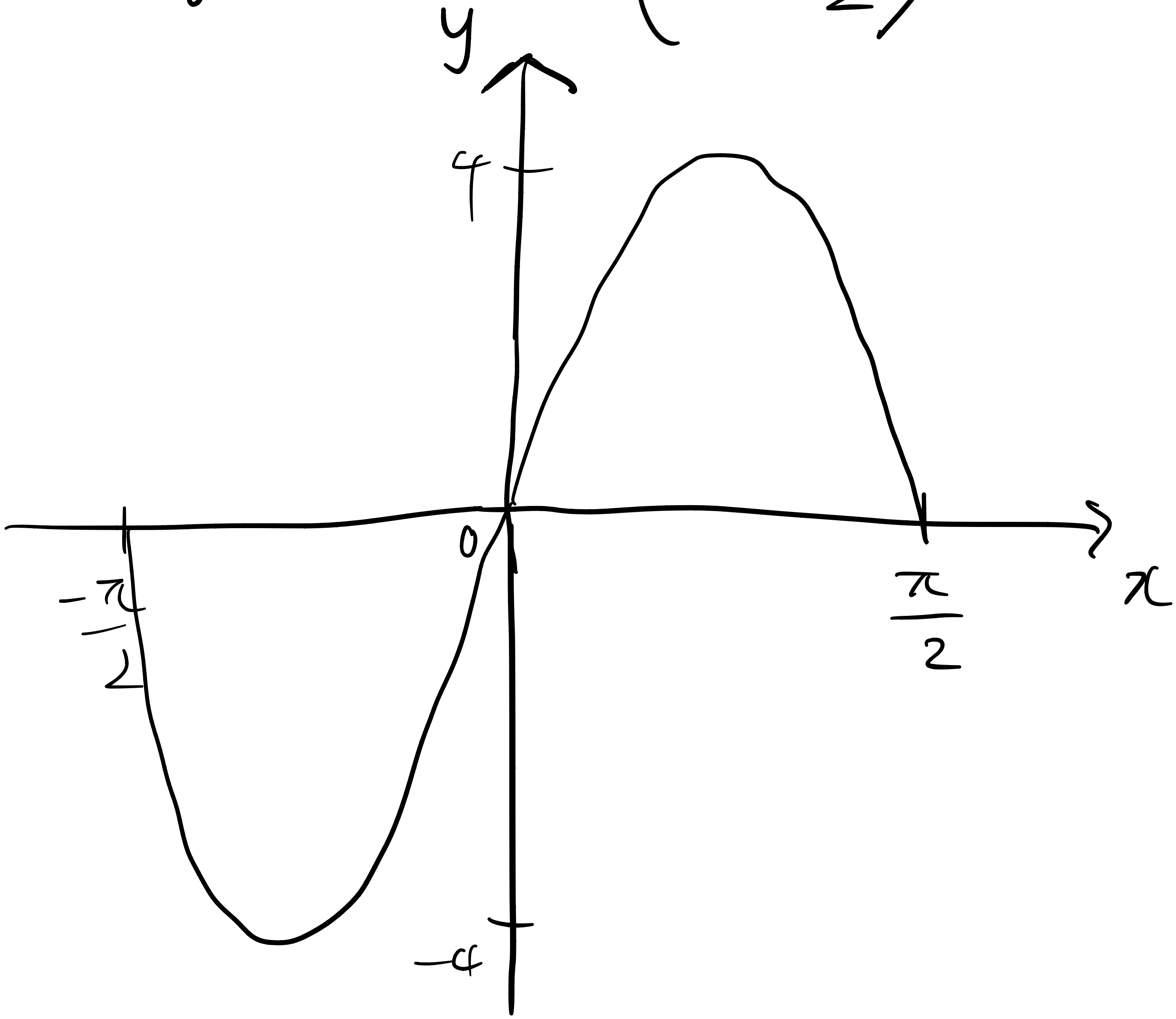
25. $y = 10 \sin \frac{1}{2} x$



35. $y = -2\sin\left(x - \frac{\pi}{6}\right)$



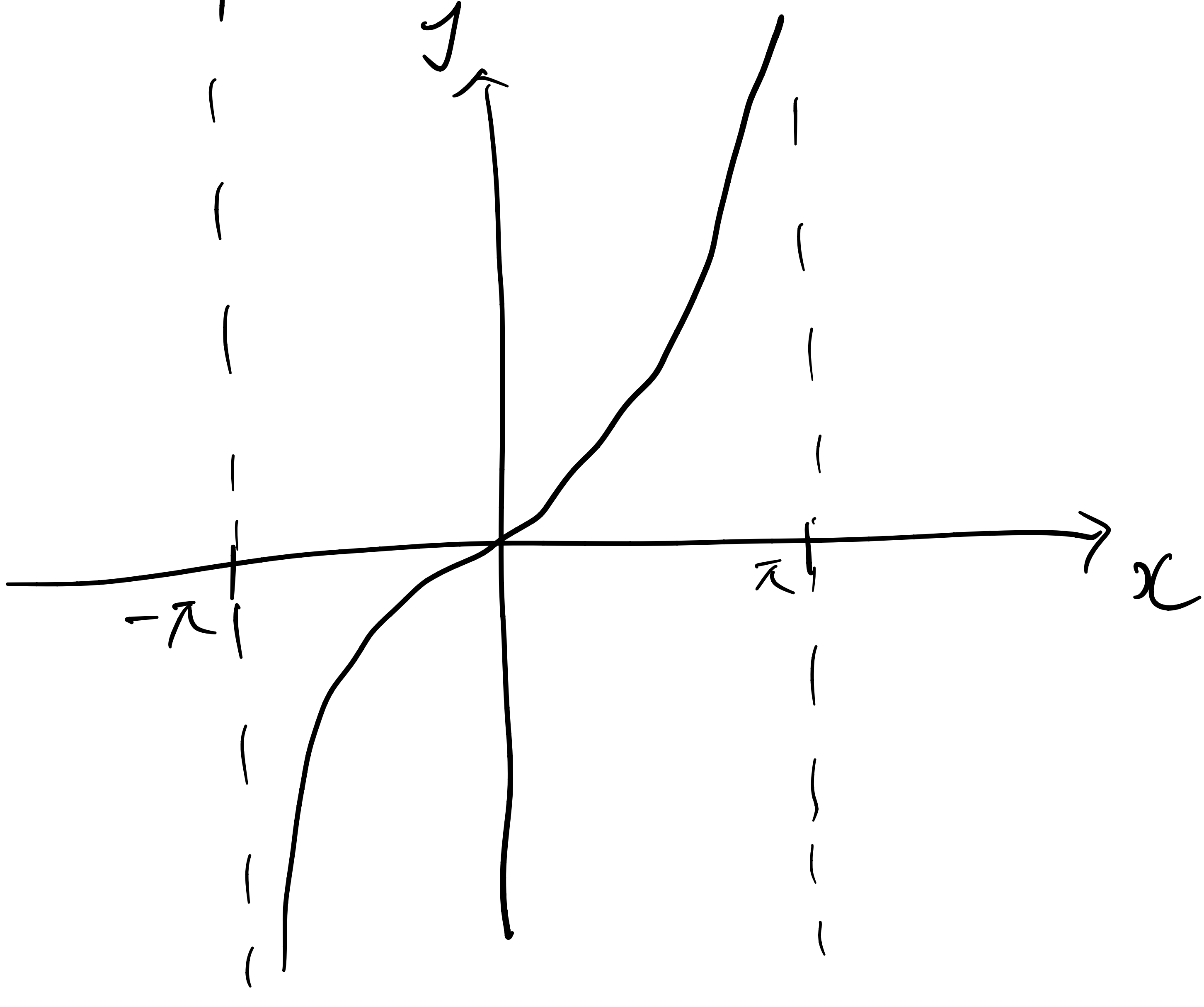
37. $y = -4\sin 2\left(x + \frac{\pi}{2}\right)$



6.4

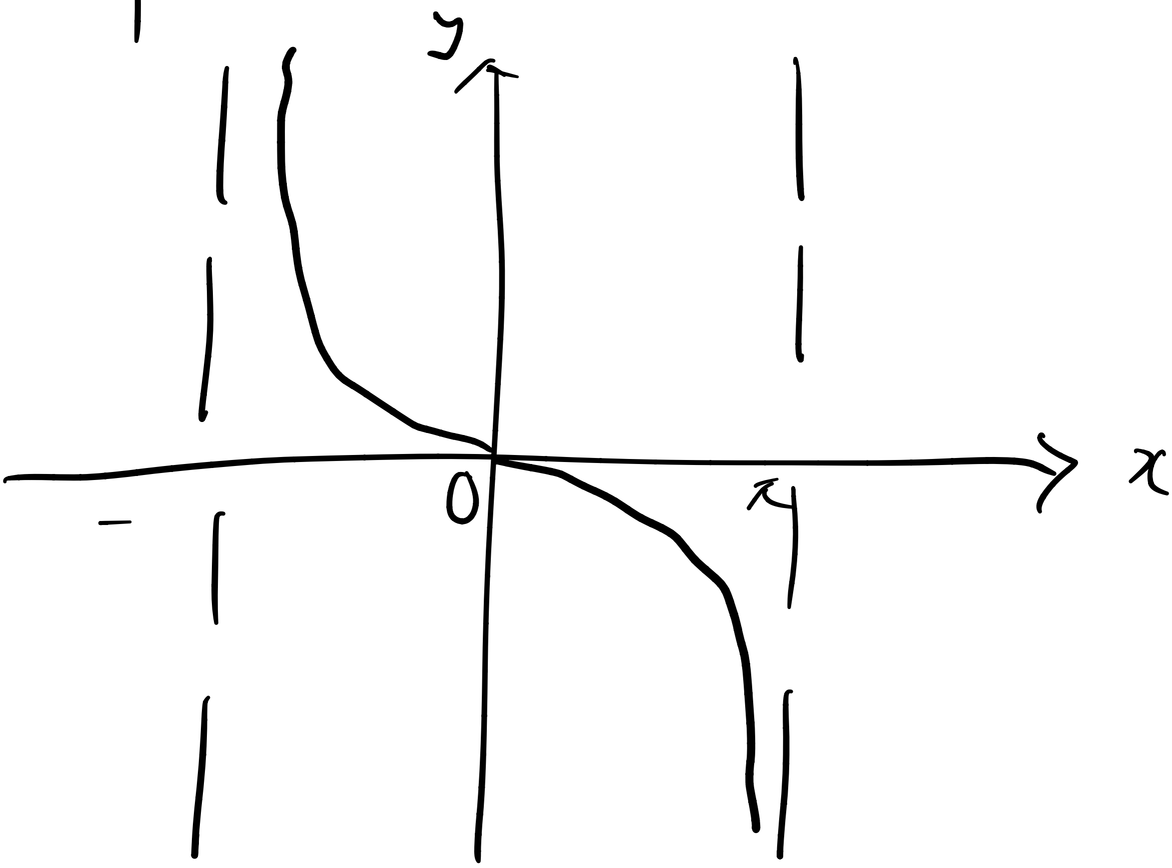
9. $y = 3\tan x$

period $= \pi$



11. $y = -\frac{3}{2}\tan x$

period = π



19. $y = \tan 3x$

6.5

3. (a) $\sin^{-1} 1$

$= \frac{\pi}{2}$

(b) $\sin^{-1} \frac{\sqrt{3}}{2}$

$= \frac{\pi}{3}$

(c) $\sin^{-1} 2$

$\therefore$ undefined

5. (a) $\cos^{-1}(-1)$

$= \pi$

(b) $\cos^{-1} \frac{1}{2}$

$= \frac{\pi}{3}$

(c) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{5\pi}{6}$

7. (a) $\tan^{-1}(-1)$

$= -\frac{\pi}{4}$

(b) $\tan^{-1}\sqrt{3}$

$= \frac{\pi}{3}$

(c) $\tan^{-1} \frac{\sqrt{3}}{3}$

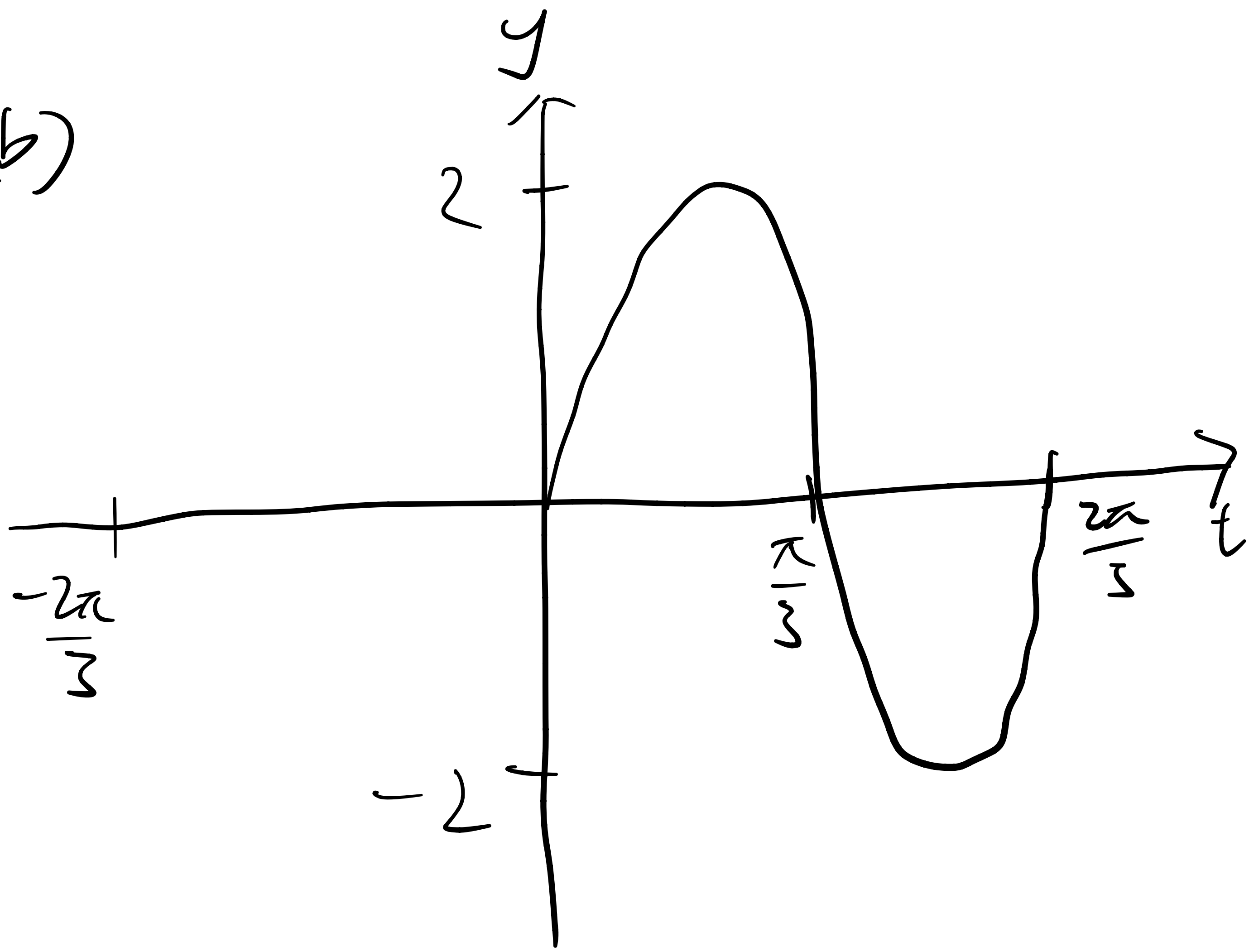
$= \frac{\pi}{6}$

6. 6

5. $y = 2\sin 3t$

(a) $A = 2$, $T = \frac{2\pi}{3}$, $f = \frac{3}{2\pi}$

(b)



Chapter 6 Exercises

3. $t = \frac{2\pi}{3}$

(a) $\hat{t} = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

(b) $P(x, y)$

$= P(-\cos\hat{t}, \sin\hat{t})$

$= P\left(-\cos\frac{\pi}{3}, \sin\frac{\pi}{3}\right)$

$= P\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

(c) $\sin t = \sin\hat{t}$

$= \sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

$\cos t = -\cos\hat{t}$

$= -\cos\frac{\pi}{3}$

$= -\frac{1}{2}$

$\tan t = -\tan\hat{t}$

$= -\tan\frac{\pi}{3} = -\sqrt{3}$

$\csc\theta = \frac{2\sqrt{3}}{3}$

$\sec\theta = -2$

$\cot\theta = -\frac{\sqrt{3}}{3}$

Values of Trigonometric Functions

7. (a) $\sin \frac{3\pi}{4}$

$\bar{t} = \frac{\pi}{4}$

$\sin t = \sin \bar{t}$
$= \sin \frac{\pi}{4}$
$= \frac{\sqrt{2}}{2}$

(b) $\cos \frac{3\pi}{4}$

$\bar{t} = \frac{\pi}{4}$

$\cos t = -\cos \bar{t}$
$= -\cos \frac{\pi}{4}$
$= -\frac{\sqrt{2}}{2}$

8. (a) $\tan \frac{\pi}{3}$

$= \frac{\sqrt{3}}{1}$
$= \sqrt{3}$

(b) $\tan\left(-\frac{\pi}{3}\right)$

$= -\tan \frac{\pi}{3}$
$= -\sqrt{3}$

9. (a) $\sin 1.1$

10. (a) $\cos\frac{\pi}{5}$

(b) $\cos\left(-\frac{\pi}{5}\right)$

$= \cos\left(\frac{\pi}{5}\right)$

17. $$\frac{\tan t}{\cos t} = \frac{\sin t}{\cos^2 t}$$

$$= \frac{\sin t}{1-\sin^2 t}$$

18. $\tan^2 t \sec t$

$$= \left(\frac{\sin t}{\cos t}\right)^2 \left(\frac{1}{\cos t}\right)$$

$$= \frac{\sin^2 t}{\cos^3 t}$$

$$= \frac{1-\cos^2 t}{\cos^3 t}$$

$\cos^2 t + \sin^2 t = 1$

$\cos t = \pm\sqrt{1-\sin^2 t}$

19. $$\tan t = \frac{\sin t}{\cos t}$$

$$= \frac{\sin t}{\pm\sqrt{1-\sin^2 t}}$$

$$= \frac{\sin t}{\sqrt{1-\sin^2 t}}$$

20. $\sec t = \dfrac{1}{\cos t}$

$$= \frac{1}{\pm\sqrt{1 - \sin^2 t}}$$

$$= -\frac{1}{\sqrt{1 - \sin^2 t}}$$

21. $\sin t = \frac{5}{13}$, $\cos t = -\frac{12}{13}$

$\tan t = -\frac{5}{12}$ $\cot t = -\frac{12}{5}$

$\csc t = \frac{13}{5}$

$\sec t = -\frac{13}{12}$

22. $\sin t = -\frac{1}{2}$, $\cos t > 0$

$\tan t = \frac{1}{\sqrt{3}}$ $\cot = \sqrt{3}$

$\sec t = \frac{2}{\sqrt{3}}$

$\csc t = -2$

$\cos t = \frac{\sqrt{3}}{2}$

25. $\sec t + \cot t$, $\tan t = \frac{1}{4}$

$\sec t + \cot t$

$= \frac{1}{\cos t} + \frac{1}{\tan t}$

$= \frac{1}{\cos t} + \frac{\cos t}{\sin t}$

$= -\sqrt{17} + 4$

$\tan t = \frac{1}{4}$

$\frac{\sin t}{\cos t} = \frac{1}{4}$

$\frac{\cos t}{\sin t} = 4$

$\frac{\sin t}{\cos t} = \frac{1}{4}$

$\sin t = \frac{\cos t}{4}$

$\sin^2 t + \cos^2 t = 1$

$1 + \cot^2 t = \sec^2 t$

$1 + 4^2 = \sec^2 t$

$\sec^2 t = 17$

$\sec t = -\sqrt{17}$

29. $y = 10\cos\frac{1}{2}x$

(a) a : 10

$w = \frac{1}{2}$ rad s^{-1}

$f = \frac{w}{2\pi} = \frac{\frac{1}{2}}{2\pi}$

$= \frac{1}{4\pi}$

$= 0.07958$

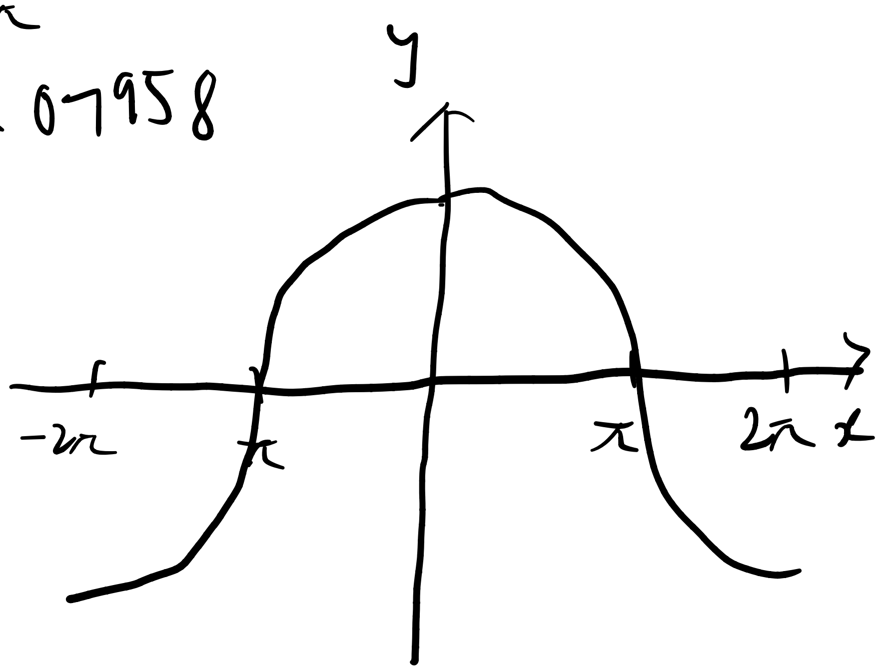
$\therefore T = \frac{2\pi}{w}$

$= \frac{2\pi}{\frac{1}{2}}$

$= 4\pi$

Horizontal shift : 0

(b)



31. $y = -\sin\frac{1}{2}x$

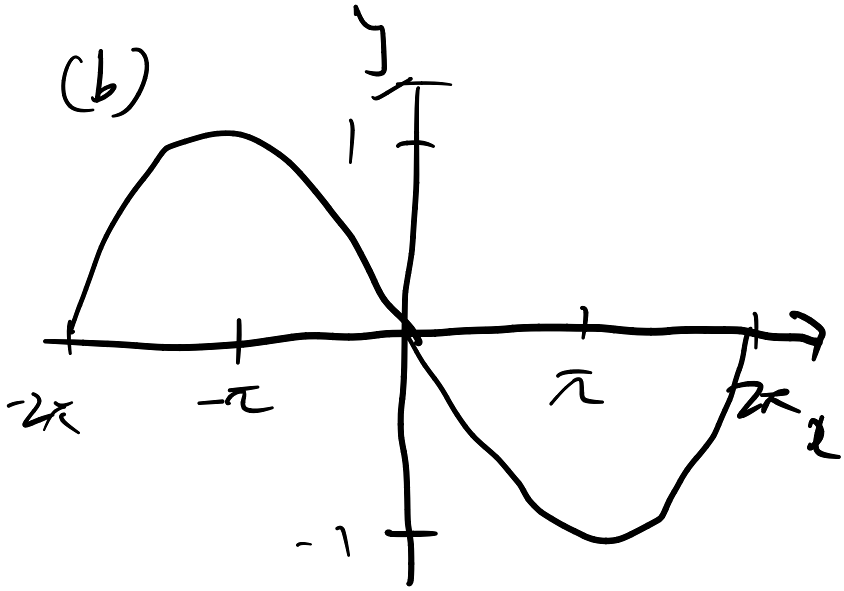
(a) a: 1

$f = \dfrac{\frac{1}{2}}{2\pi} = \dfrac{1}{4\pi}$

$\therefore T = 4\pi$

Horizontal shift: 0

(b)



33. $y = 3\sin(2x-2)$

(a) a: 3

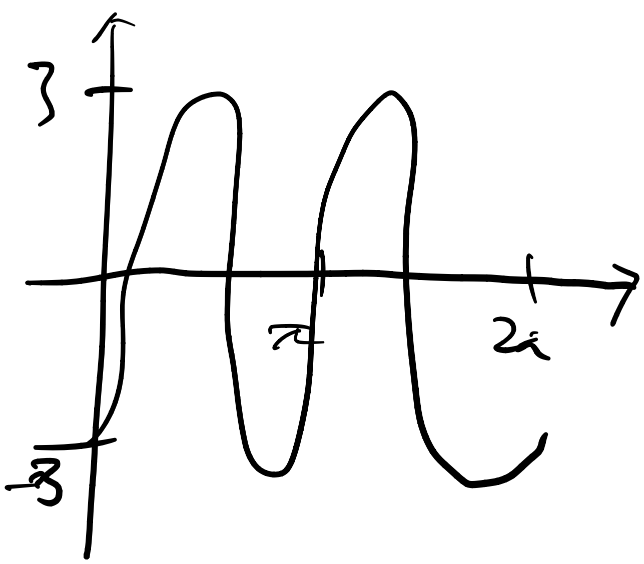
$\omega = 2\ rad\ s^{-1}$

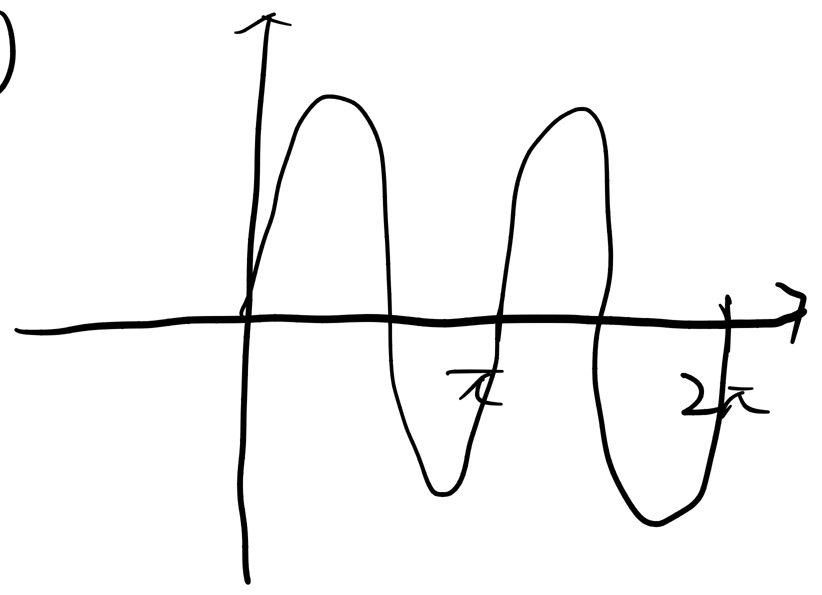
$f = \dfrac{2}{2\pi} = \dfrac{1}{\pi}\ s^{-1}$ one / second

$\therefore T = \pi$ second / unit

Horizontal shift: delayed 2 seconds

(b)





31. $y = -\sin \frac{1}{2}x$

(a) a: 1

$w = \frac{1}{2} \text{ rad s}^{-1}$

$f = \frac{w}{2\pi}$

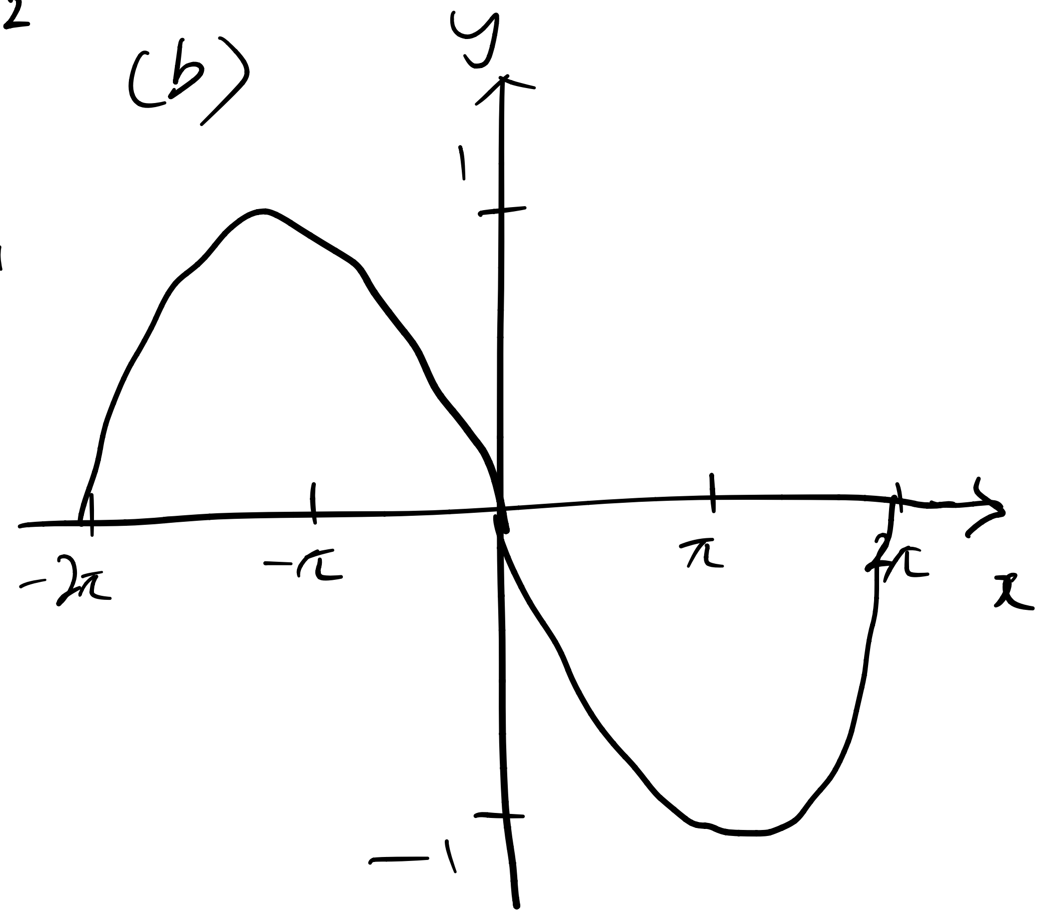
$= \frac{\frac{1}{2}}{2\pi}$

$= \frac{1}{4\pi}$

$T = 4\pi$

h: 0

(b)



33. $y = 3\sin(2x-2)$

$= 3\sin(2(x-1))$

(a)

a: 3

$w = 2 \text{ rad s}^{-1}$

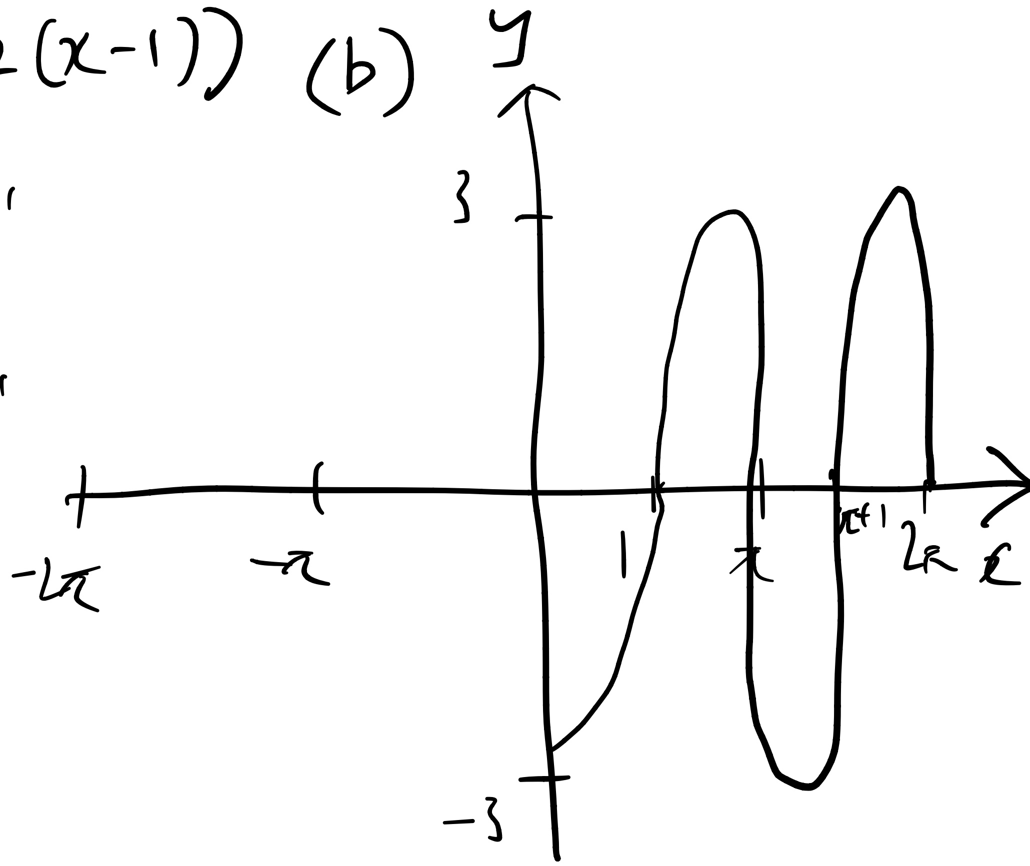
$f = \frac{2}{2\pi}$

$= \frac{1}{\pi} \text{ s}^{-1}$

T: π

h: delayed 1

(b)



35. $y = -\cos\left(\frac{\pi}{2}x + \frac{\pi}{6}\right) = -\cos\left(\frac{\pi}{2}\left(x + \frac{1}{3}\right)\right)$

(a) $a : 1$

$\omega = \frac{\pi}{2}$ rad s^{-1}

$f = \frac{\frac{\pi}{2}}{2\pi} = \frac{1}{4} s^{-1}$

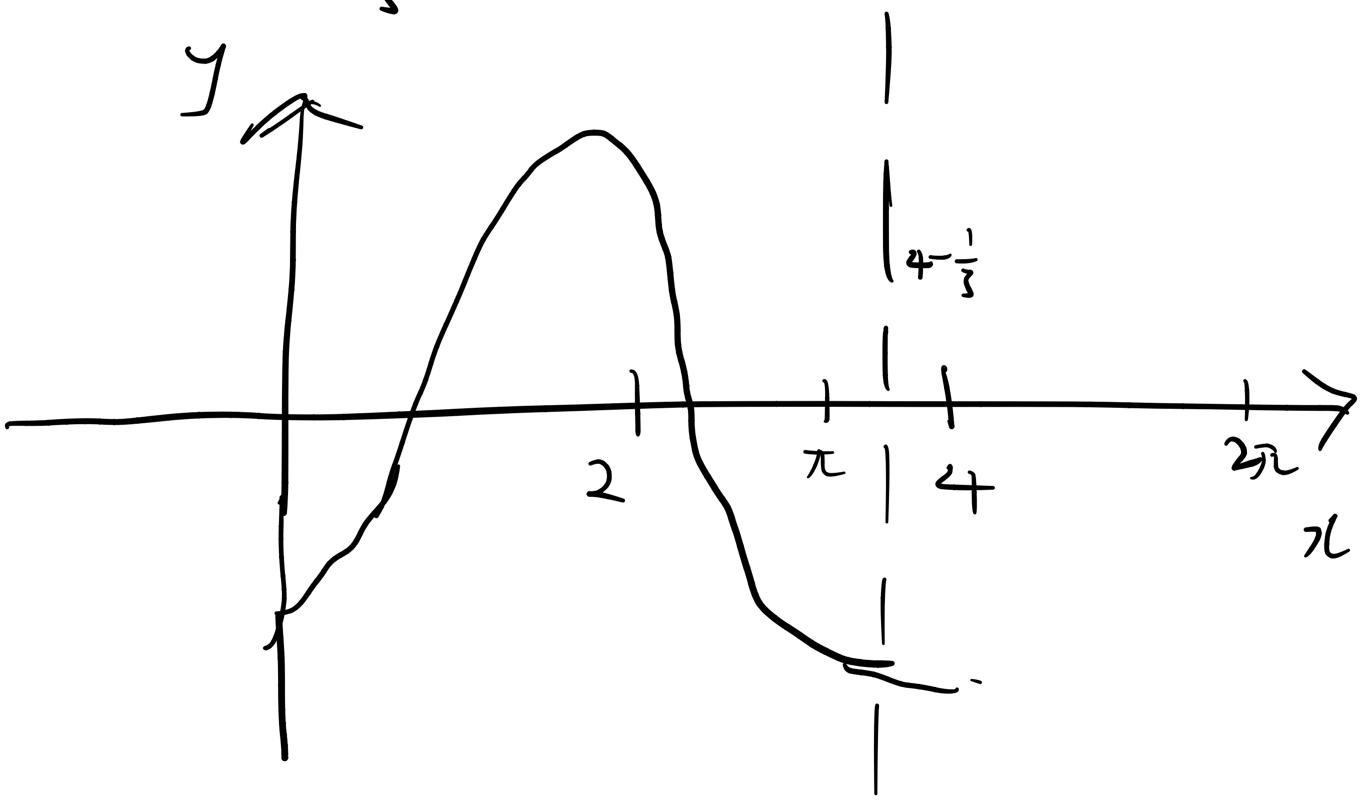
$T : 4s$

h : ahead $\frac{1}{3}$

$\frac{\pi}{4} \times \overset{1}{\cancel{2}}\pi = \pi \times \pi$

$= 1.56 \times 1.56$

(b)



37. $T = \frac{\pi}{2}$

$f = \frac{2}{\pi}$

$w = 2\pi\left(\frac{2}{\pi}\right)$

$= 4$

$\therefore 5 \sin 4x$

40. $T = \frac{4\pi}{3}$

$f = \frac{3}{4\pi}$

$= 0.2387$

$w = \cancel{2\pi}\left(\frac{3}{\cancel{4\pi}}\right)$

$= 1.5 = \frac{3}{2}$

$\therefore 4 \cos \frac{3}{2}x$

$4 \sin\left(\frac{\pi}{2} - \frac{3}{2}x\right)$

$= 4 \sin$

38. $T = 4$

$f = \frac{1}{4}$ unit time

$w = 2\pi\left(\frac{1}{4}\right)$

$= \frac{\pi}{2}$

$\therefore 2 \sin \frac{\pi}{2}x$

39. $h : \frac{1}{5}$ $f = 1$

$w = 2\pi(1) = 2\pi$

$T = 1$

$\therefore \frac{1}{2} \sin 2\pi\left(x + \frac{1}{3}\right)$

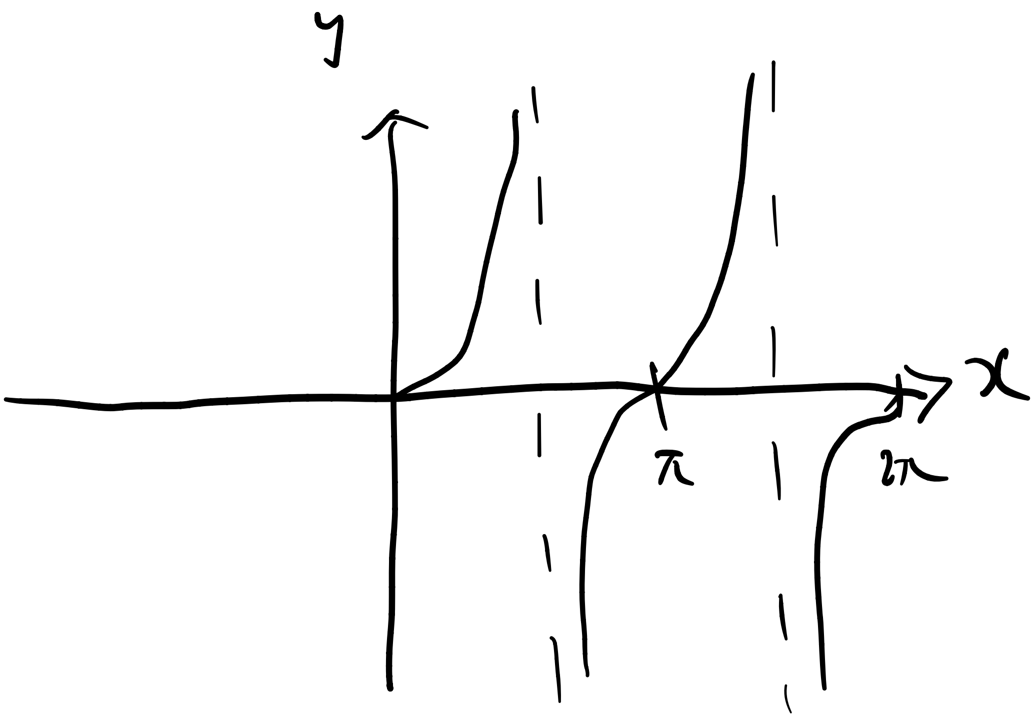
41. $y = 3\tan x$

$\omega = 1$ rad / unit time

$f = \frac{1}{\pi}$

$= 0.318$ / unit time

$T = \pi$ unit time

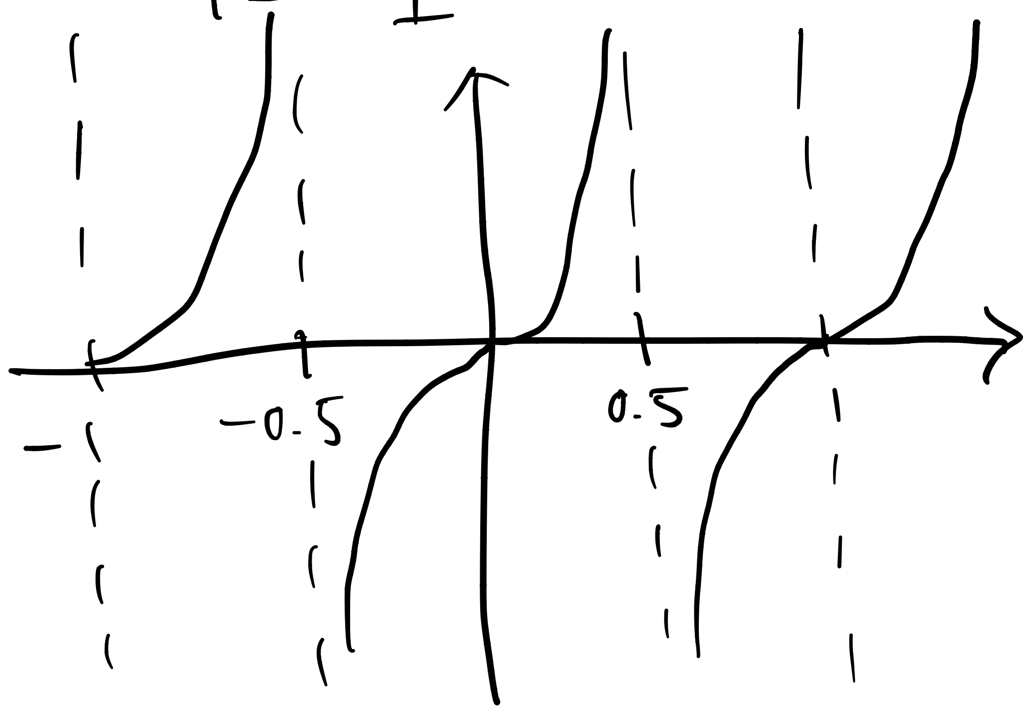


42. $y = \tan \pi x$

$\omega = \pi$ rad / unit time

$f = \frac{\pi}{\pi} = 1$ / unit time

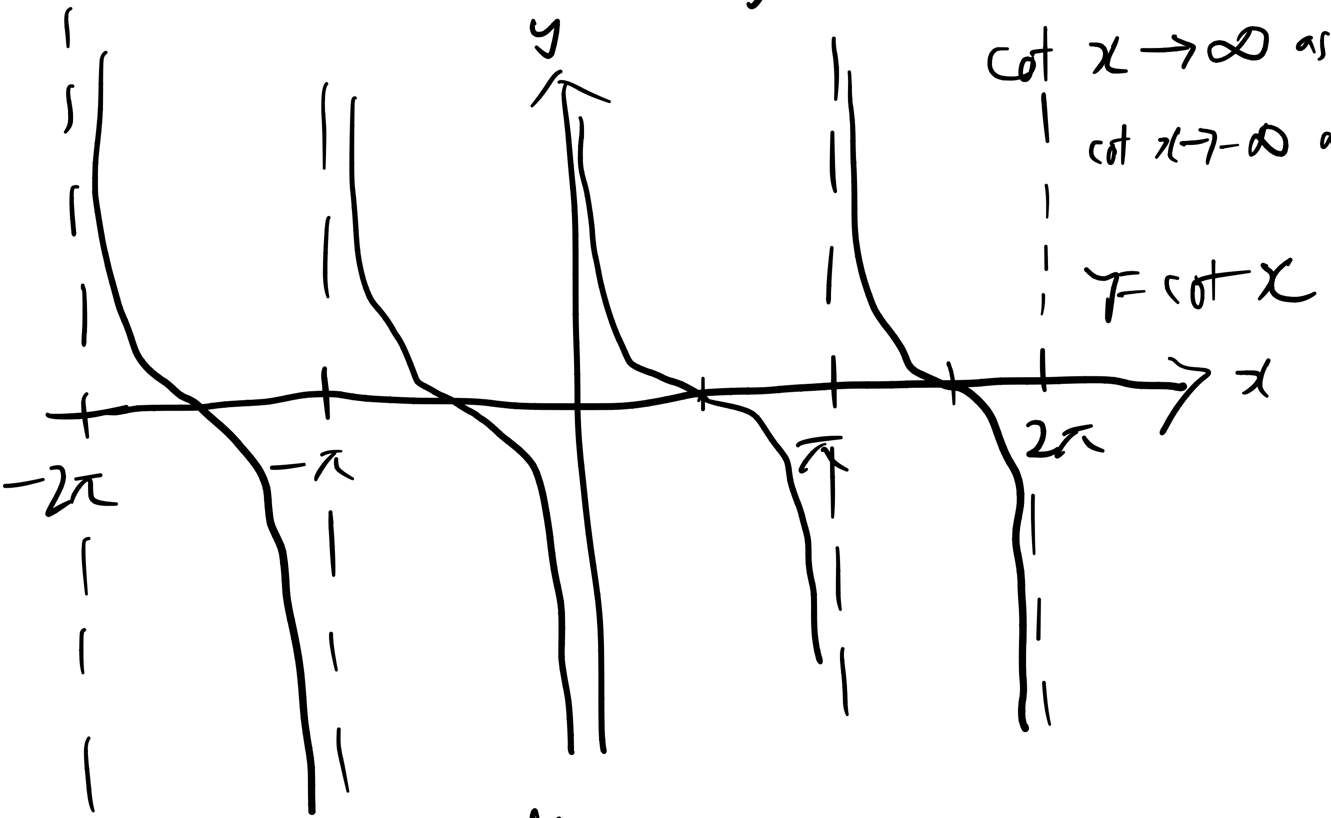
$T = 1$ unit time

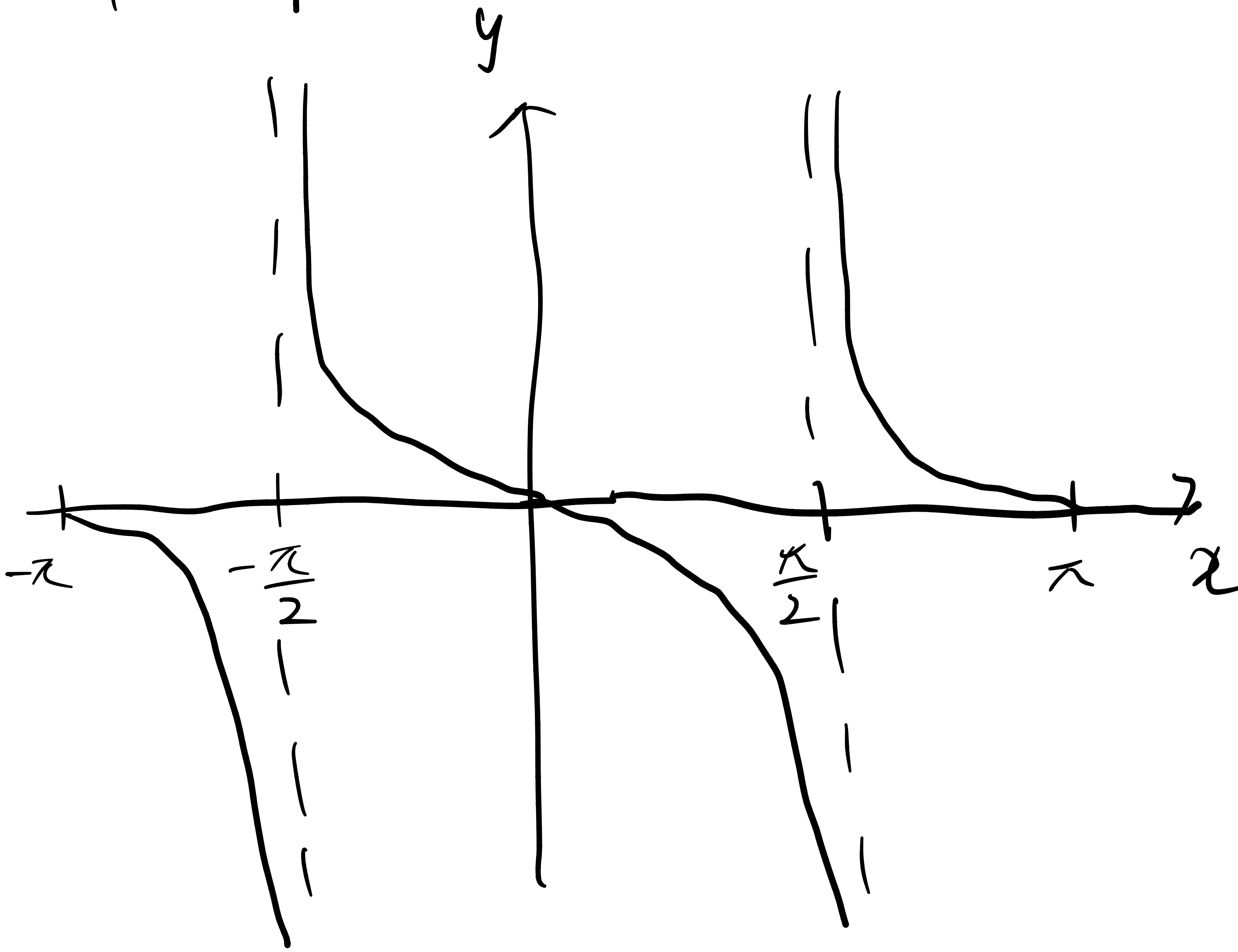


43. $y = 2\cot\left(x - \frac{\pi}{2}\right)$ $\quad T = \frac{\pi}{1} = \pi$ unit time

$\cot x \to \infty$ as $x \to 0^+$

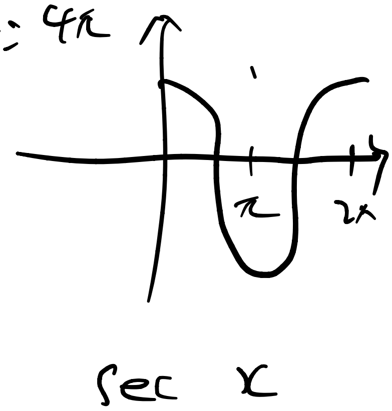
$\cot x \to -\infty$ as $x \to \pi^-$

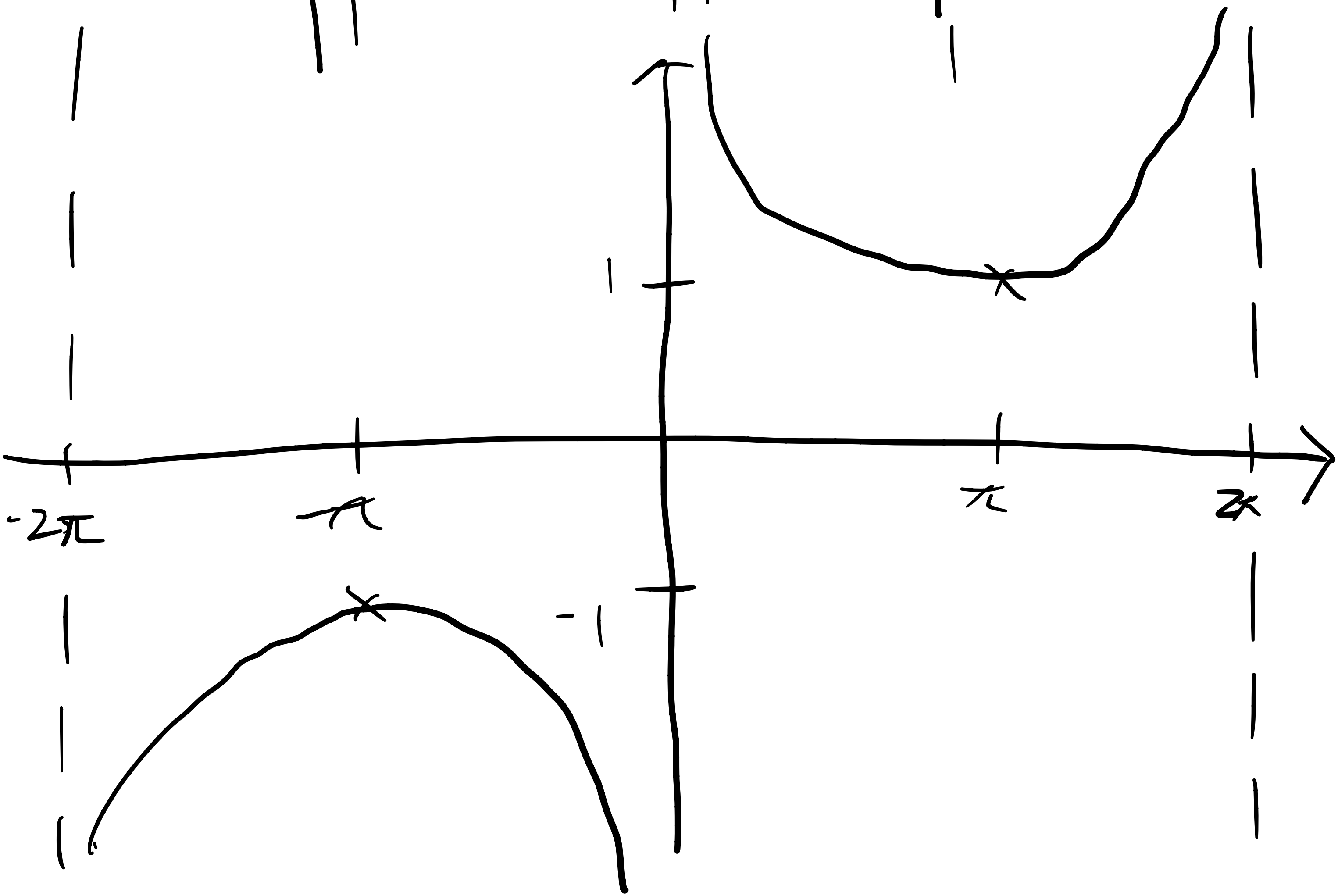




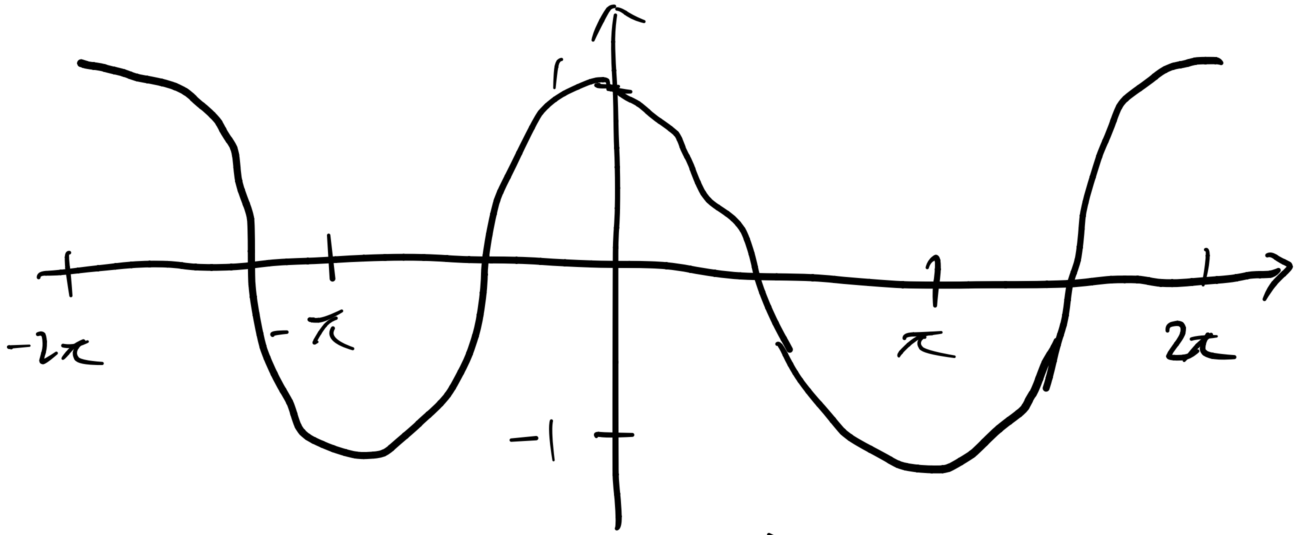
44. $y = \sec\left(\frac{1}{2}x - \frac{\pi}{2}\right)$ $T = \frac{2\pi}{\frac{1}{2}} = 4\pi$

$= \sec \frac{1}{2}(x - \pi)$

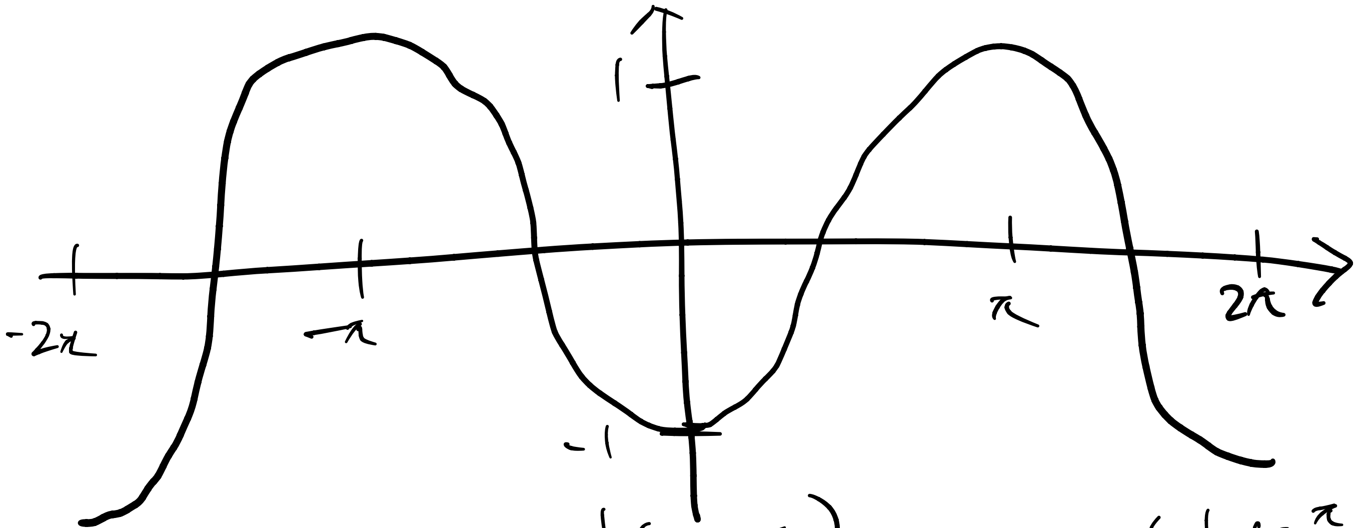




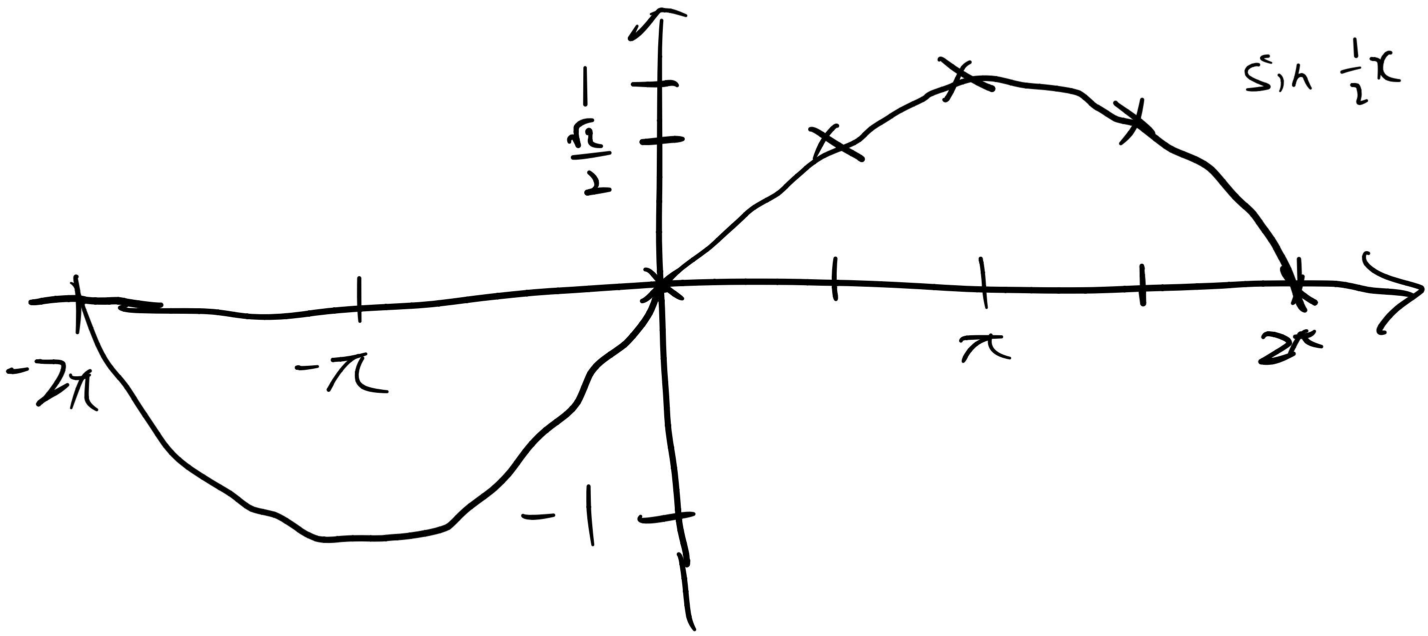
$\cos x$



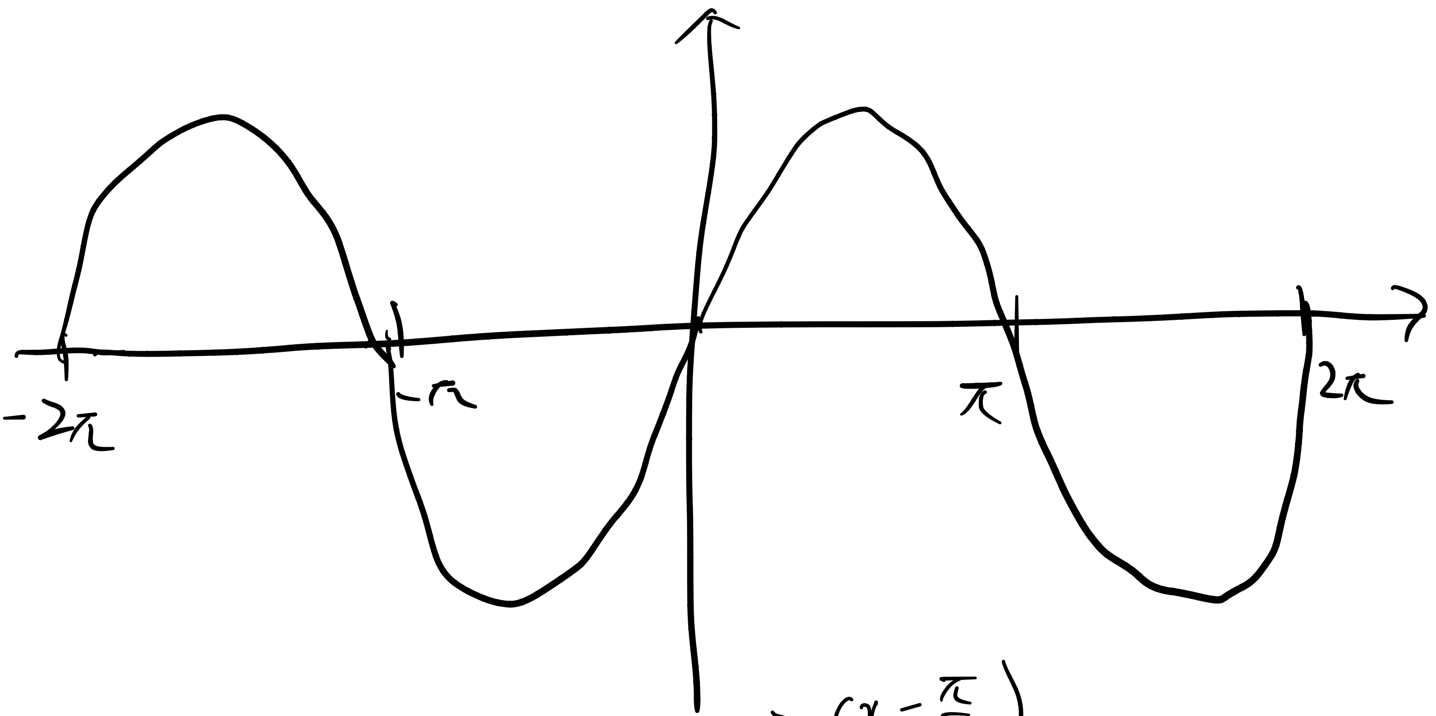
$\cos(x-\pi)$



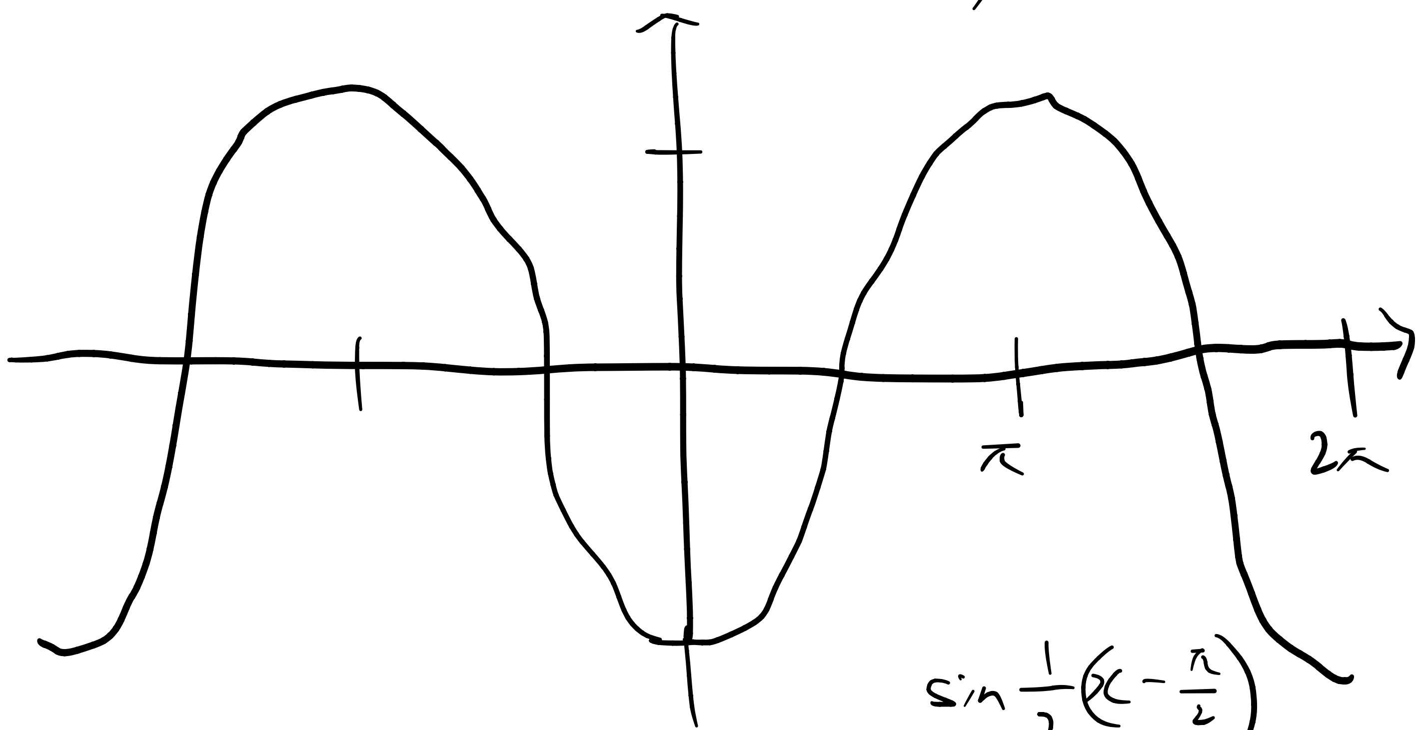
$\cos \frac{1}{2}(x-\pi)$ $\cos\left(\frac{1}{2}x-\frac{\pi}{2}\right)$



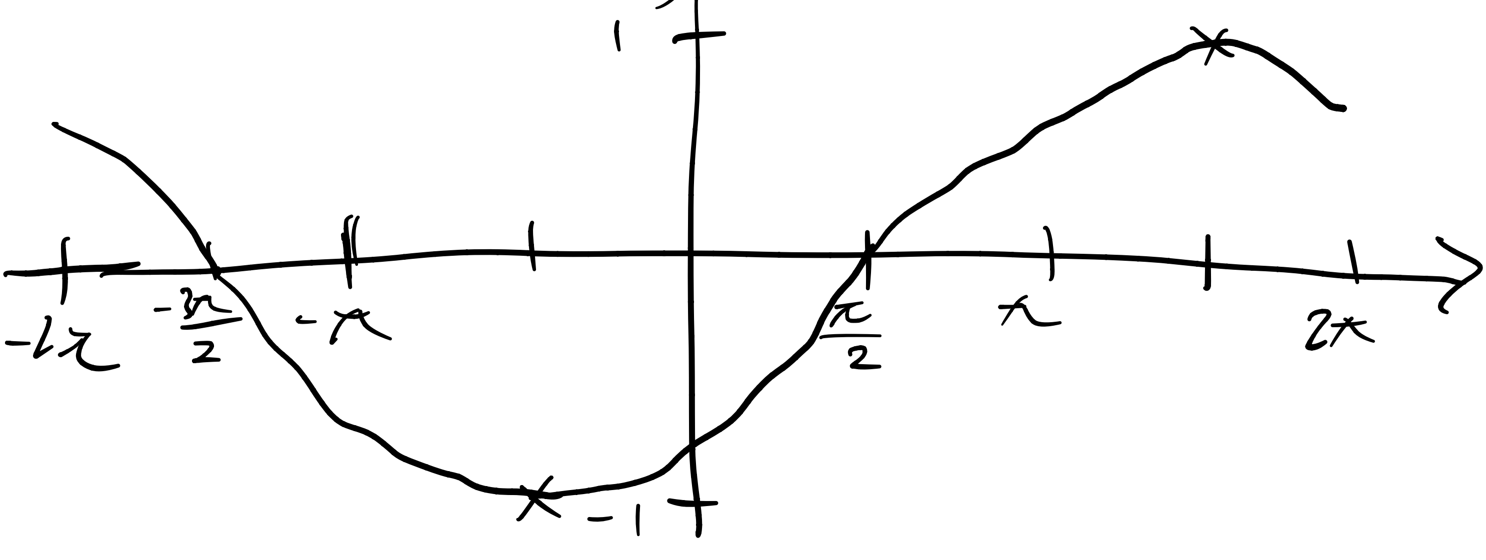
$\sin x$



$\sin\left(x - \frac{\pi}{2}\right)$



$\sin \frac{1}{2}\left(x - \frac{\pi}{2}\right)$



45. $y = 4\csc(2x+\pi)$

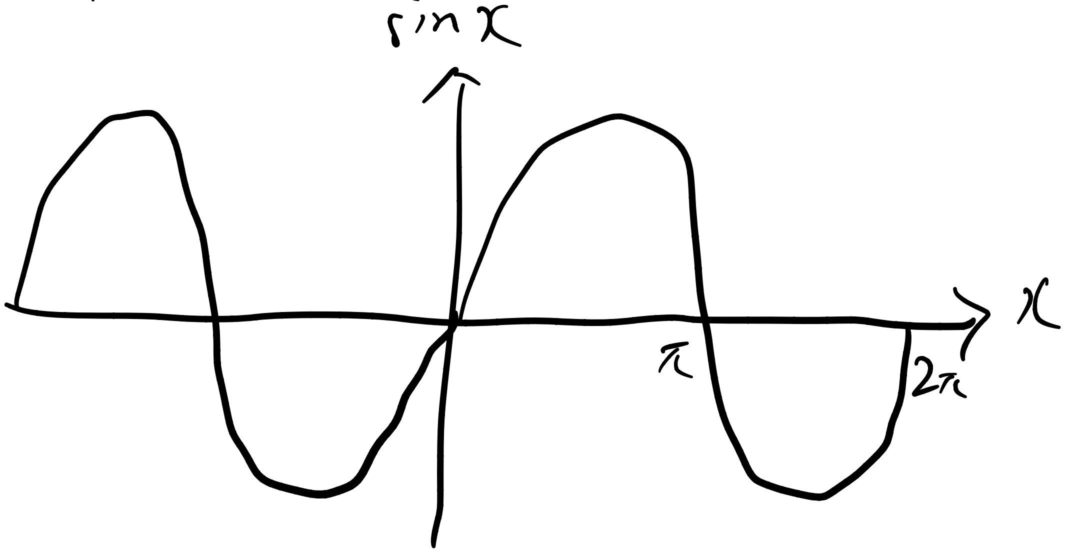
$y = 4\csc 2\left(x+\frac{\pi}{2}\right)$

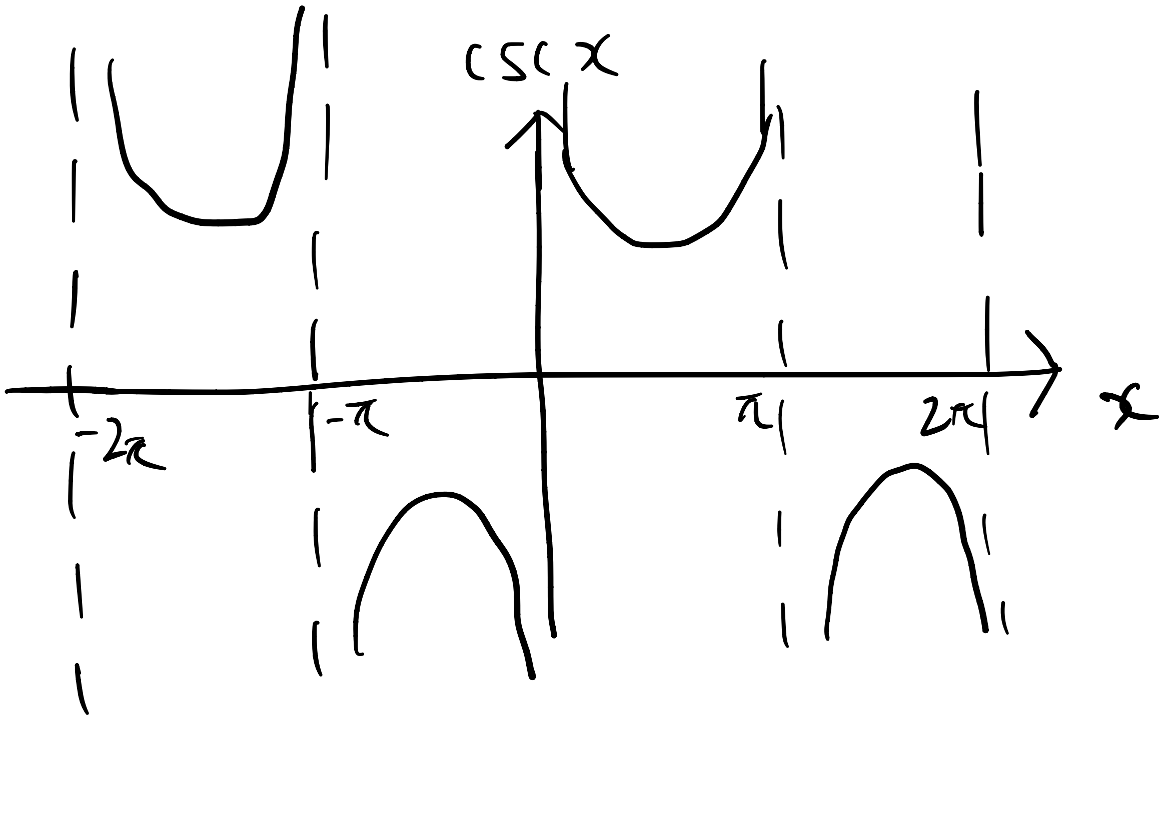
$\omega = 2$ rad / unit time

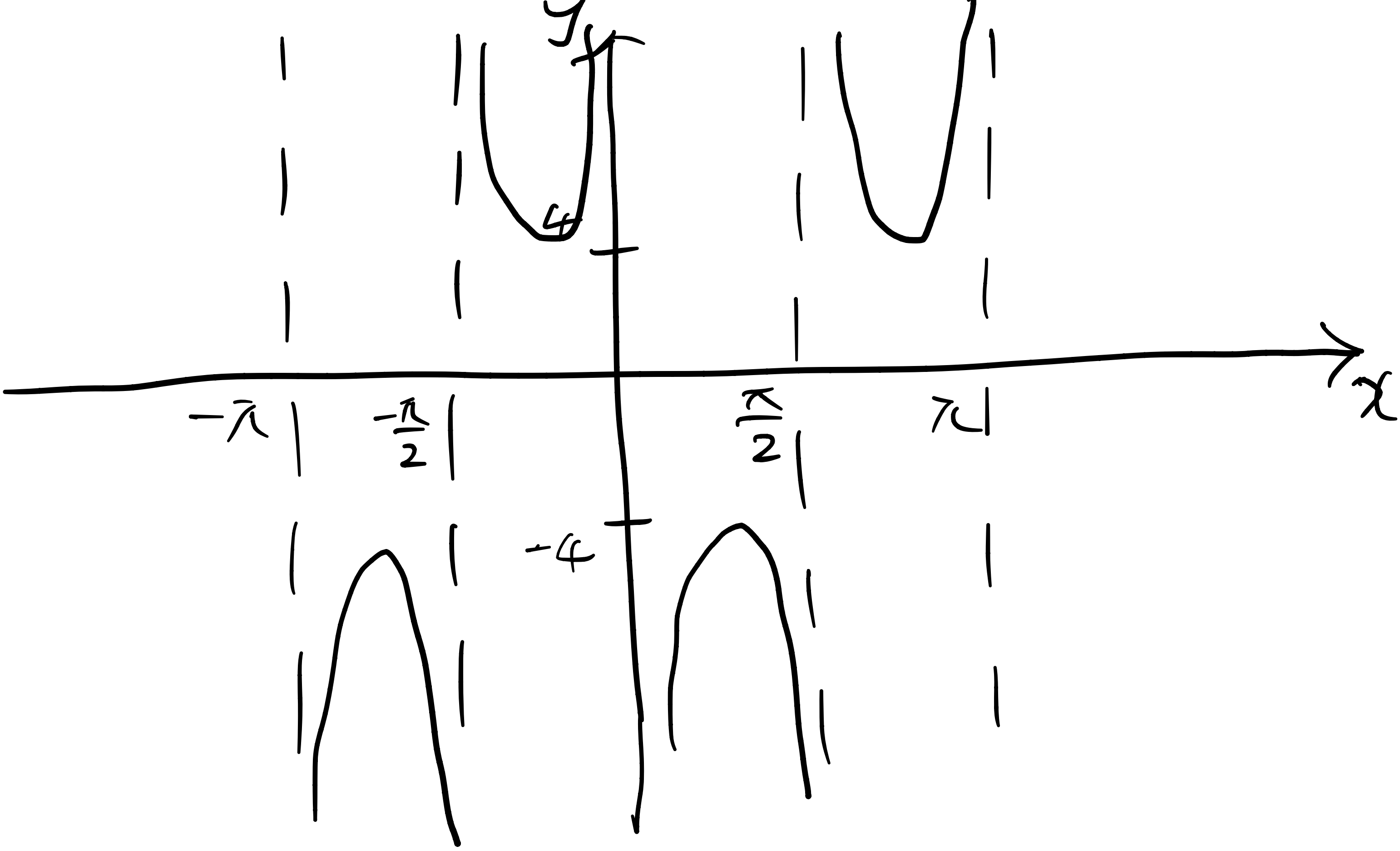
$f = \frac{2}{2\pi}$

$= \frac{1}{\pi}$ unit/unit time

$T = \pi$ unit time/ unit





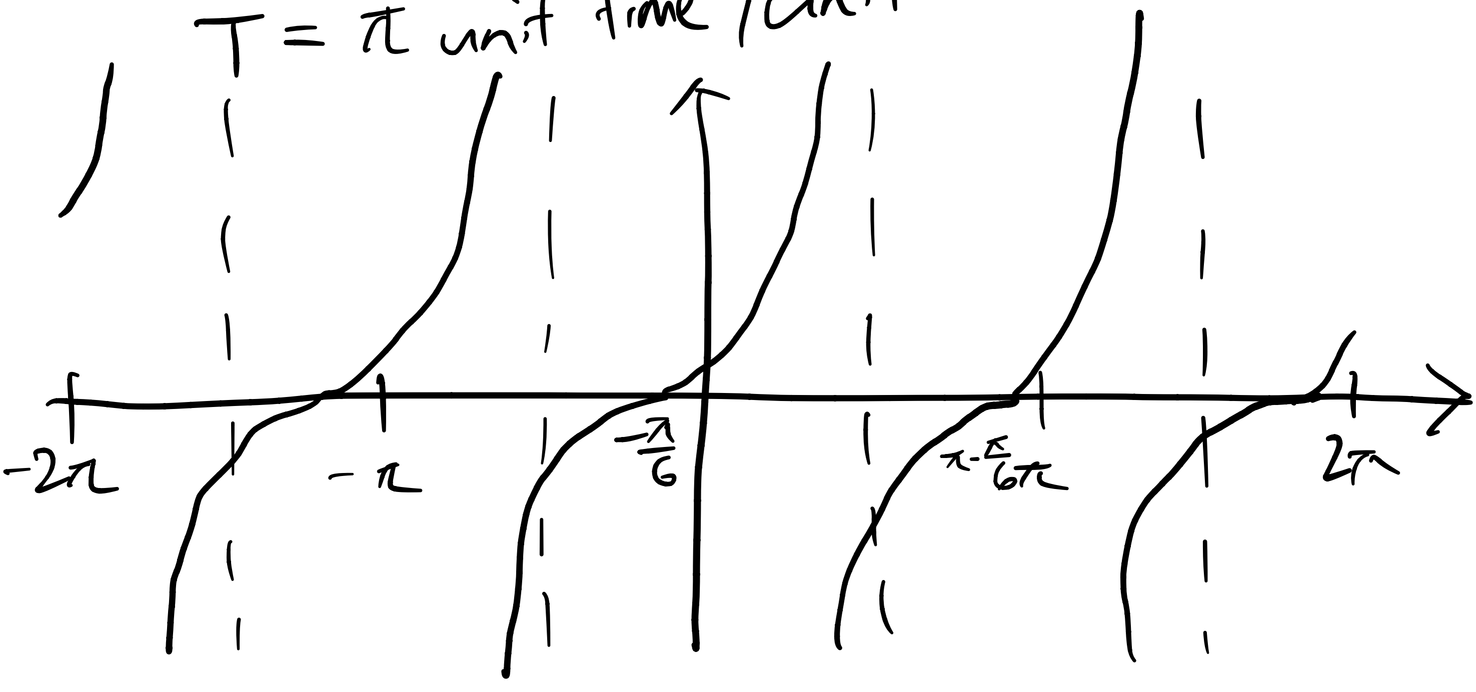


46. $y = \tan\left(x + \frac{\pi}{6}\right)$

$w = 1$

$f = \frac{1}{\pi}$ unit/unit time

$T = \pi$ unit time/unit



49. $\sin^{-1} 1 = \dfrac{\pi}{2}$

50. $\cos^{-1}\left(-\dfrac{1}{2}\right)$

$\bar{\phi} = \dfrac{\pi}{3}$

$\phi = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

51. $\sin^{-1}\left(\sin \dfrac{13\pi}{6}\right) = \dfrac{13\pi}{6} = \dfrac{\pi}{6}$

52. $\tan\left(\cos^{-1}\left(\dfrac{1}{2}\right)\right)$

$= \tan\left(\dfrac{\pi}{3}\right)$

$= \sqrt{3}$

53. $y = 100 \sin 8\left(t + \frac{\pi}{16}\right)$

a: 100

$w = 8$ rad/unit time

$f = \frac{8}{2\pi}$

$= \frac{4}{\pi}$ unit/unit time

T: $\frac{\pi}{4}$ unit time/unit

ϕ: $8\left(-\frac{\pi}{16}\right) = -\frac{\pi}{2}$

HS: $\frac{\pi}{16}$ to the left

54. $y = 80 \sin 3\left(t - \frac{\pi}{2}\right)$

a: 80

T: $\frac{2\pi}{3}$

ϕ: $\frac{3\pi}{2}$

HS: $\frac{\pi}{2}$ to the right

Chapter 7

3. $\cos t \tan t$

$$= \cos t \frac{\sin t}{\cos t}$$

$$= \sin t$$

23. $\dfrac{1+\sin u}{\cos u} + \dfrac{\cos u}{1+\sin u}$

$$= \frac{(1+\sin u)^2 + (\cos u)^2}{(1+\sin u)(\cos u)}$$

$$= \frac{1+2\sin u+\sin^2 u+\cos^2 u}{\cos u+\sin u\cos u}$$

$$= \frac{1+2\sin u+1}{\cos u+\sin u\cos u}$$

$$= \frac{2+2\sin u}{\cos u(1+\sin u)}$$

$$= \frac{2(1+\sin u)}{\cos u(1+\sin u)}$$

$$= \frac{2}{\cos u}$$

29. $$\frac{\cos x}{\sec x \sin x} = \csc x - \sin x$$

(a)

(b)

LHS:

$$\frac{\cos x}{\frac{1}{\cos x} \sin x}$$

$$= \frac{\cos^2 x}{\sin x}$$

$$= \frac{1 - \sin^2 x}{\sin x}$$

$$= \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$$

$$= \csc x - \sin x$$

$\therefore$ LHS = RHS

65. $\dfrac{1-\cos x}{\sin x} + \dfrac{\sin x}{1-\cos x} = 2\csc x$

LHS: $\dfrac{(1-\cos x)^2 + \sin^2 x}{(1-\cos x)(\sin x)}$

$= \dfrac{1-2\cos x+\cos^2 x+\sin^2 x}{(1-\cos x)(\sin x)}$

$= \dfrac{2-2\cos x}{(1-\cos x)(\sin x)}$

$= \dfrac{2(1-\cos x)}{\sin x(1-\cos x)}$ $\therefore$ LHS = RHS

$= \dfrac{2}{\sin x}$

$= 2\csc x$

77. $\dfrac{1-\cos\alpha}{\sin\alpha} = \dfrac{\sin\alpha}{1+\cos\alpha}$

$(1-\cos\alpha)(1+\cos\alpha) = (\sin\alpha)^2$

$1^2 - \cos^2\alpha = \sin^2\alpha$

LHS: $1^2 - \cos^2\alpha$

$= 1-\cos^2\alpha = \sin^2\alpha$ $\therefore$ LHS = RHS

83. $\dfrac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta$

RHS: $\sec\theta + \tan\theta$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1+\sin\theta}{\cos\theta}$

$\dfrac{\cos\theta}{1-\sin\theta} = \dfrac{1+\sin\theta}{\cos\theta}$

$\cos^2\theta = 1^2 - \sin^2\theta$

RHS: $1-\sin^2\theta = \cos^2\theta$

$\therefore$ LHS = RHS

89. $\dfrac{x}{\sqrt{1-x^2}}$, $x = \sin\theta$, $0 < \theta < \frac{\pi}{2}$

$$\frac{x}{\sqrt{1-x^2}}$$

$$= \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{\sin\theta}{\sqrt{\cos^2\theta}}$$

$\cos\theta$ is positive

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

7.2

3. $\sin 75^\circ$

$= \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$

9. $\sin \frac{19\pi}{12} = \sin\left(\frac{4\pi}{3} + \frac{\pi}{4}\right)$

$\sin\frac{4\pi}{3} = -\sin\frac{\pi}{3}$

$= -\frac{\sqrt{3}}{2}$

$t = \frac{\pi}{3}$

$= \sin\frac{4\pi}{3}\cos\frac{\pi}{4} + \cos\frac{4\pi}{3}\sin\frac{\pi}{4}$

$= -\frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{6}}{4}$

$= -\frac{\sqrt{6}}{2}$

15. $\sin 18^\circ \cos 27^\circ + \cos 18^\circ \sin 27^\circ$

$$= \sin(18^\circ + 27^\circ)$$

$$= \sin 45^\circ$$

$$= \frac{\sqrt{2}}{2}$$

21. $\tan\left(\frac{\pi}{2} - u\right) = \cot u$

LHS: $\tan\left(\frac{\pi}{2} - u\right)$

$$= \frac{\sin\left(\frac{\pi}{2} - u\right)}{\cos\left(\frac{\pi}{2} - u\right)}$$

$$= \frac{\sin\frac{\pi}{2}\cos u - \cos\frac{\pi}{2}\sin u}{\cos\frac{\pi}{2}\cos u + \sin\frac{\pi}{2}\sin u}$$

$$= \frac{1 \cdot \cos u - 0 \cdot \sin u}{0 \cdot \cos u + 1 \cdot \sin u}$$

$$= \frac{\cos u}{\sin u}$$

$$= \cot u$$

25. $\sin\left(x-\frac{\pi}{2}\right)=-\cos x$

LHS: $\sin\left(x-\frac{\pi}{2}\right)$

$=\sin x\cos\frac{\pi}{2}-\cos x\sin\frac{\pi}{2}$

$=\sin x\cdot 0-\cos x\cdot 1$

$=-\cos x$

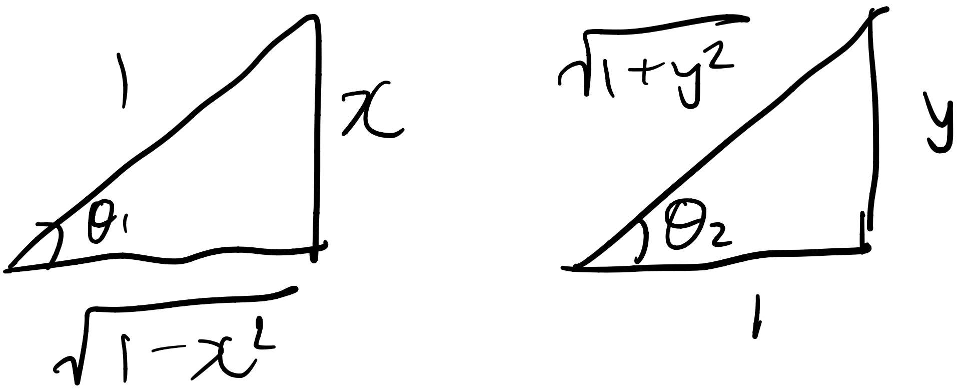
33. $\tan\left(x+\frac{\pi}{3}\right)=\dfrac{\tan x+\tan\frac{\pi}{3}}{1-\tan x\tan\frac{\pi}{3}}$

$=\dfrac{\tan x+\sqrt{3}}{1-\tan x\sqrt{3}}$

$=\dfrac{\sqrt{3}+\tan x}{1-\sqrt{3}\tan x}$

47. $\cos(\sin^{-1}x - \tan^{-1}y)$

Let $\sin^{-1}x = \theta_1$, $\tan^{-1}y = \theta_2$



$$\cos(\theta_1 - \theta_2)$$

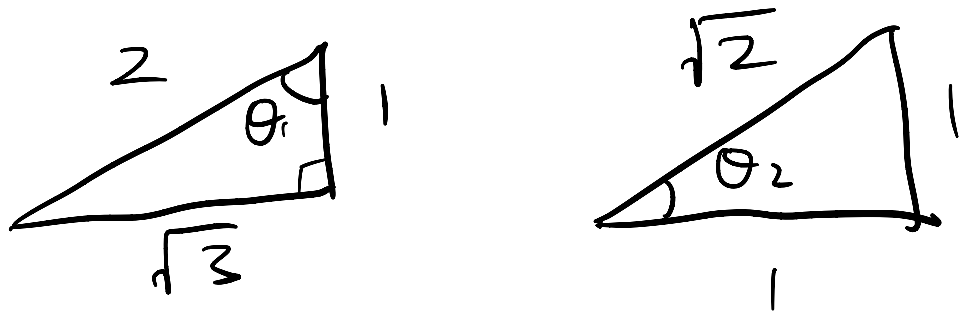
$$= \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2$$

$$= \sqrt{1-x^2}\,\frac{1}{\sqrt{1+y^2}} + x\,\frac{y}{\sqrt{1+y^2}}$$

$$= \frac{\sqrt{1-x^2} + xy}{\sqrt{1+y^2}}$$

51. $\sin\left(\cos^{-1}\frac{1}{2} + \tan^{-1} 1\right)$

Let $\cos^{-1}\frac{1}{2} = \theta_1$, $\tan^{-1} 1 = \theta_2$



$= \sin(\theta_1 + \theta_2)$

$= \sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2$

$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$

$= \frac{\sqrt{6} + \sqrt{2}}{4}$

55. $\cos(\theta - \phi)$, $\cos\theta = \frac{3}{5}$, Q IV

$\tan\phi = -\sqrt{3}$, Q II

$\cos(\theta - \phi)$

$= \cos\theta\cos\phi + \sin\theta\sin\phi$

$\cos\phi = -\frac{1}{2}$

$= \frac{3}{5}\left(-\frac{1}{2}\right) + \left(-\frac{4}{5}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\sin\theta = -\frac{4}{5}$

$= -\frac{3}{10} - \frac{4\sqrt{3}}{10}$

$= \frac{-3 - 4\sqrt{3}}{10}$

59.

$$-\sqrt{3}\sin x+\cos x$$

$$=-\sqrt{3}\sin x+\cos x$$

$$=-2\left(\frac{\sqrt{3}}{2}\sin x-\frac{1}{2}\cos x\right)$$

$$=-2\left(\cos\frac{\pi}{6}\sin x-\sin\frac{\pi}{6}\cos x\right)$$

$$=-2\sin\left(x-\frac{\pi}{6}\right)$$

$$=2\sin\left(-\left(x-\frac{\pi}{6}\right)\right)$$

$$=2\sin\left(-x+\frac{\pi}{6}\right)$$

63. $g(x) = \cos 2x + \sqrt{3}\sin 2x$

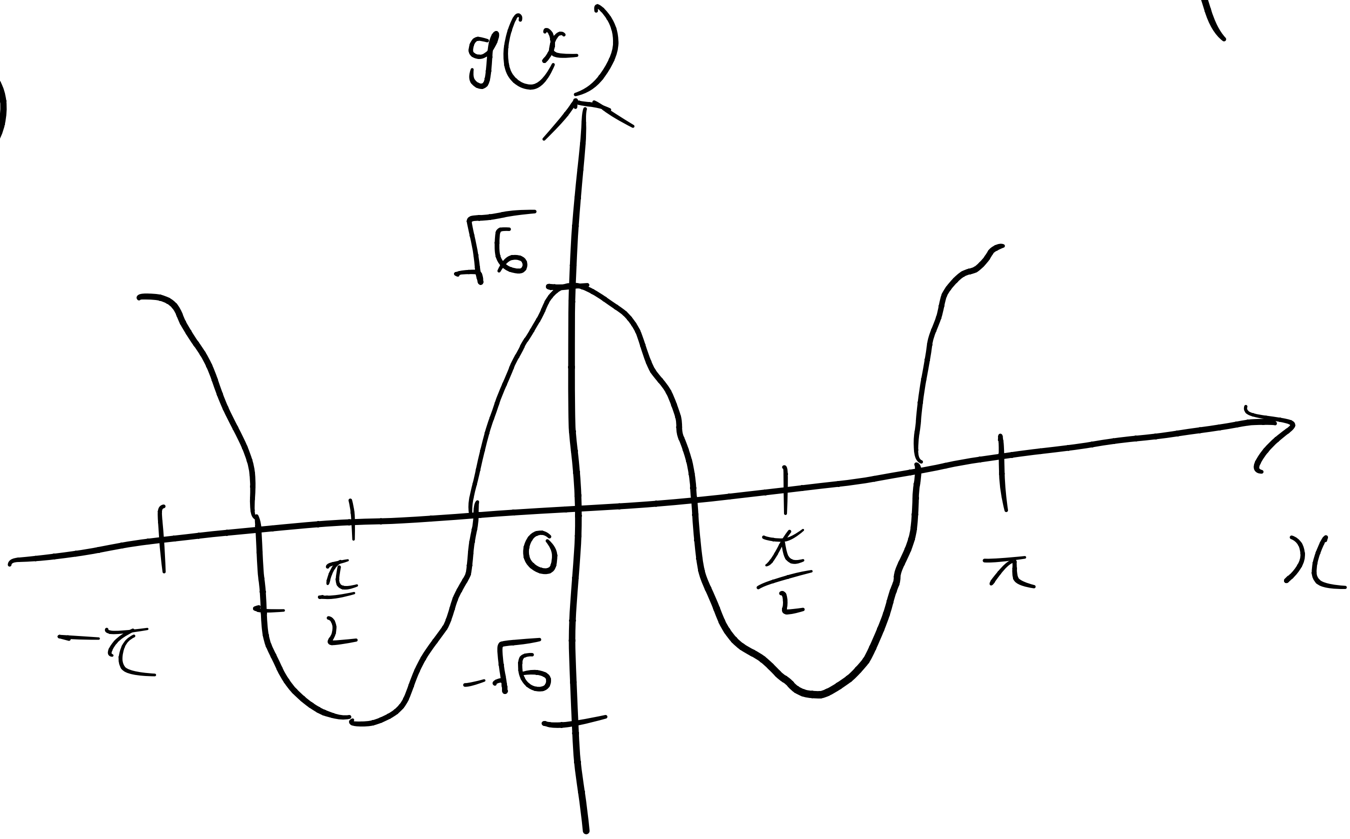
(a) $g(x) = \sqrt{3}\sin 2x + \cos 2x$

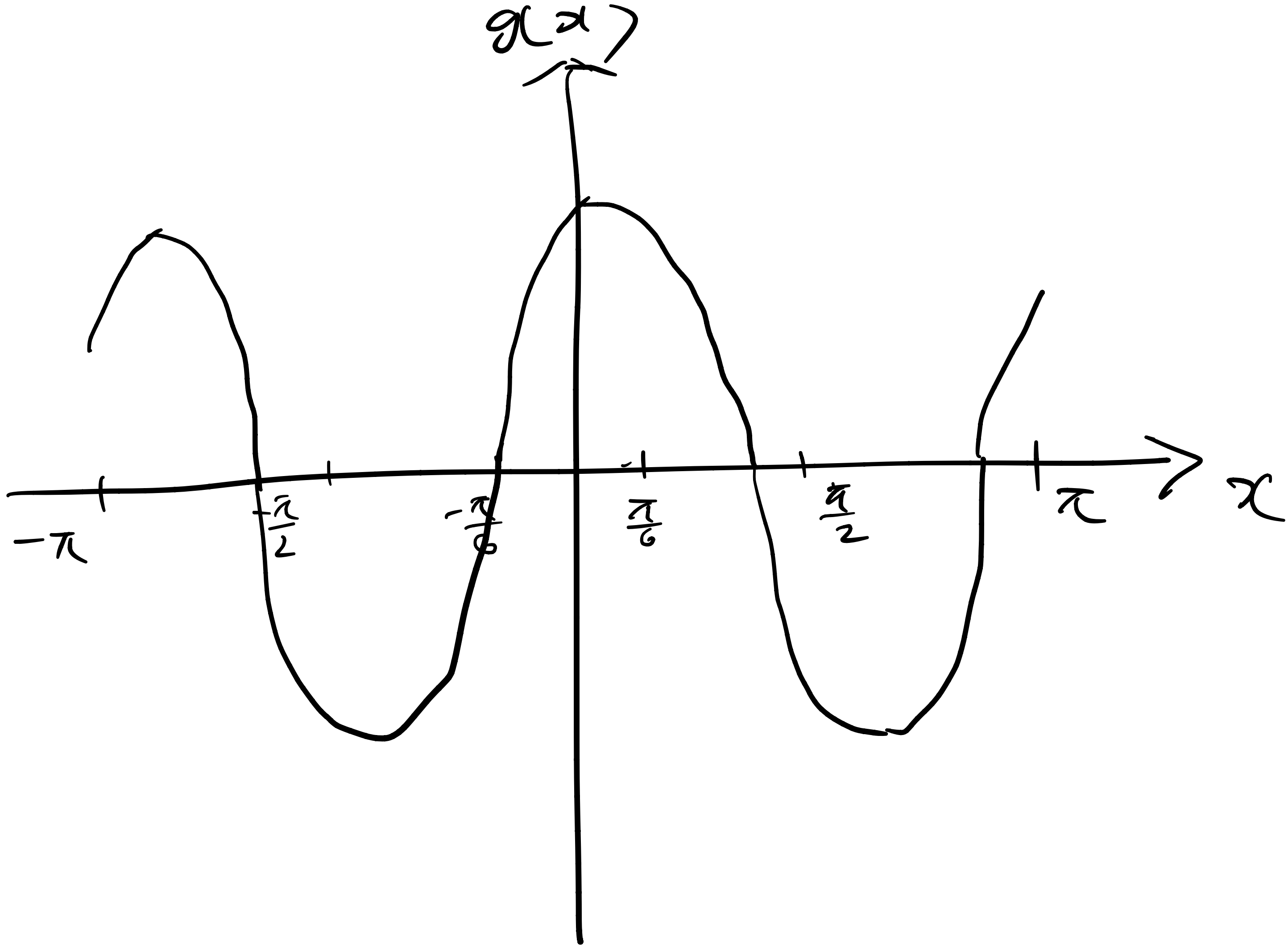
$= 2\left(\frac{\sqrt{3}}{2}\sin 2x + \frac{1}{2}\cos 2x\right)$

$= 2\left(\cos\frac{\pi}{6}\cos 2x + \sin\frac{\pi}{6}\sin 2x\right)$

$= 2\cos\left(2x - \frac{\pi}{6}\right)$

(b)




g(x)
x
-π
-π/2
-π/6
π/6
π/2
π

65. $$\frac{f(x+h)-f(x)}{h} = -\cos x\left(\frac{1-\cos h}{h}\right) - \sin x\left(\frac{\sin h}{h}\right)$$

$f(x) = \cos x$

num:

$\cos(x+h) - \cos x$

$= \cos x \cos h - \sin x \sin h - \cos x$

$$\frac{f(x+h)-f(x)}{h}$$

$$= \frac{\cos(x+h) - \cos x}{h}$$

$$= \frac{-\cos x(1-\cos h) - \sin x \sin h}{h}$$

$$= -\cos x\left(\frac{1-\cos h}{h}\right) - \sin x\left(\frac{\sin h}{h}\right)$$

$\therefore$ shown

17. $\sin(x+y) = \sin x \cos y + \cos x \sin y$

$$
\begin{aligned}
&\sin(x+y) \\
&= \cos\left(\frac{\pi}{2} - (x+y)\right) \\
&= \cos\left(\left(\frac{\pi}{2} - x\right) - y\right) \\
&= \cos\left(\frac{\pi}{2} - x\right)\cos y + \sin\left(\frac{\pi}{2} - x\right)\sin y \\
&= \sin x \cos y + \cos x \sin y
\end{aligned}
$$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$$
\boxed{
\begin{aligned}
\sin(x-y) &= \sin(x+(-y)) \\
&= \sin x \cos(-y) + \cos x \sin(-y) \\
&= \sin x \cos y - \cos x \sin y
\end{aligned}
}
$$

78. $\tan(x+y)$

$$= \frac{\sin(x+y)}{\cos(x+y)}$$

$$= \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$$

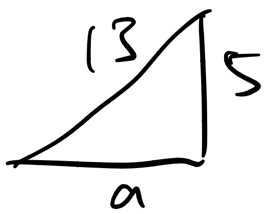
$$= \frac{\dfrac{\sin x \cancel{\cos y}}{\cos x \cancel{\cos y}} + \dfrac{\cancel{\cos x} \sin y}{\cancel{\cos x} \cos y}}{\dfrac{\cancel{\cos x \cos y}}{\cancel{\cos x \cos y}} - \dfrac{\sin x \sin y}{\cos x \cos y}}$$

$$= \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

7.3

3. $\sin x = \dfrac{5}{13}$, QI

$$\sin 2x = \sin(x+x)$$
$$= \sin x \cos x + \cos x \sin x$$
$$= 2\sin x \cos x$$
$$= 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)$$
$$= \frac{120}{169}$$



$a = \sqrt{13^2 - 5^2}$
$= 12$

$$\cos 2x = \cos(x+x)$$
$$= \cos x \cos x - \sin x \sin x$$
$$= \cos^2 x - \sin^2 x$$
$$= \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$$
$$= \frac{144-25}{169}$$
$$= \frac{119}{169}$$

$$\tan 2x = \tan(x+x)$$

$$= \frac{\tan x + \tan x}{1 - \tan x \tan x}$$

$$= \frac{2\tan x}{1 - \tan^2 x}$$

11. $\sin^4 x$

$$= (\sin^2 x)^2$$

$$= \left(\frac{1-\cos 2x}{2}\right)^2$$

$$= \frac{1-2\cos 2x+\cos^2 2x}{4}$$

$$= \frac{1-2\cos 2x+\left(\frac{1+\cos 4x}{2}\right)}{4}$$

$$= \frac{1}{4} - \frac{\cos 2x}{2} + \frac{1}{8} + \frac{\cos 4x}{8}$$

$$= \frac{3}{8} - \frac{\cos 2x}{2} + \frac{\cos 4x}{8}$$

$$= \frac{1}{2}\left(\frac{3}{4} - \cos 2x + \frac{1}{4}\cos 4x\right)$$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$2\cos^2 x - 1$

$= \cos 2x$

$\cos^2 x = \frac{1+\cos 2x}{2}$

17. $\sin 15^\circ$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$2\sin^2 x = 1 - \cos 2x$$
$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\sin 15^\circ$$
$$= \sin 15^\circ$$
$$= \pm\sqrt{\frac{1-\cos 2(15^\circ)}{2}}$$
$$= \pm\sqrt{\frac{1-\cos 30^\circ}{2}}$$
$$= \pm\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$$
$$= \pm\sqrt{\frac{2-\sqrt{3}}{4}}$$
$$= \pm\frac{\sqrt{2-\sqrt{3}}}{2}$$

37. $\sin x = \frac{3}{5}$, $0^\circ < x < 90^\circ$

$\cos x = \frac{4}{5}$

$\tan x = \frac{3}{4}$

$$\sin\frac{x}{2} = \sqrt{\frac{1-\cos 2\left(\frac{x}{2}\right)}{2}}$$

$$= \sqrt{\frac{1-\cos x}{2}}$$

$\cos 2x = 2\cos^2 x - 1$

$2\cos^2 x = 1 + \cos 2x$

$\cos x = \pm\sqrt{\frac{1+\cos 2x}{2}}$

$$= \sqrt{\frac{1-\frac{4}{5}}{2}}$$

$$= \sqrt{\frac{1}{10}}$$

$$= \frac{\sqrt{10}}{10}$$

$$\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}}$$

$$= \sqrt{\frac{\frac{5}{5}+\frac{4}{5}}{2}}$$

$$= \sqrt{\frac{9}{10}} = \frac{3\sqrt{10}}{10}$$

$$\tan\frac{x}{2} = \frac{\sqrt{\frac{1-\cos x}{2}}}{\sqrt{\frac{1+\cos x}{2}}}$$

$$= \frac{\sqrt{1-\cos x}}{\sqrt{1+\cos x}}$$

$$= \sqrt{\frac{1^2-\cos^2 x}{(1+\cos x)^2}}$$

$$= \sqrt{\frac{\sin^2 x}{(1+\cos x)^2}}$$

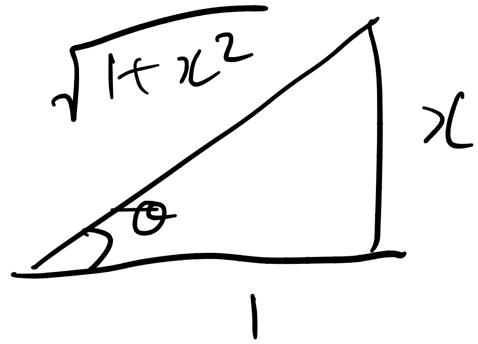
$$= \frac{+\sin x}{+(1+\cos x)}$$

$$= \frac{\sin x}{1+\cos x}$$

$$= \frac{\frac{3}{5}}{\frac{5}{5}+\frac{4}{5}}$$

$$= \frac{1}{3}$$

43. $\sin\left(2\tan^{-1}x\right)$



Let $\tan^{-1}x = \theta$,

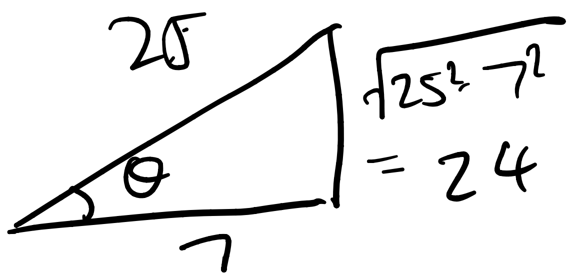
$\sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2\left(\dfrac{x}{\sqrt{1+x^2}}\right)\left(\dfrac{1}{\sqrt{1+x^2}}\right)$

$= \dfrac{2x}{1+x^2}$

47. $\sin\left(2\cos^{-1}\dfrac{7}{25}\right)$



Let $\cos^{-1}\dfrac{7}{25} = \theta$,

$\sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2\left(\dfrac{24}{25}\right)\left(\dfrac{7}{25}\right)$

$= \dfrac{336}{625}$

51. $\cos 2\theta$; $\sin\theta = -\frac{3}{5}$ QIII

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$
$$= 1 - 2\left(-\frac{3}{5}\right)^2$$
$$= 1 - \frac{18}{25}$$
$$= \frac{7}{25}$$

55. $\sin 2x \cos 3x$

$$\sin(3x+2x) = \sin 3x \cos 2x + \cos 3x \sin 2x$$

$-$

$$\sin(3x-2x) = \sin 3x \cos 2x - \cos 3x \sin 2x$$

$\downarrow$

$$= 2\cos 3x \sin 2x$$

$$\therefore \sin 2x \cos 3x = \frac{\sin(3x+2x) - \sin(3x-2x)}{2}$$

$$= \frac{\sin 5x - \sin x}{2}$$

61. $\sin 5x + \sin 3x$

$= \sin(4x + x) + \sin(4x - x)$

$\sin 5x + \sin 3x$

$= 2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}$

$= 2\sin 4x \cos x$

87. $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

LHS:

$$\tan 3x$$
$$= \tan(2x + x)$$
$$= \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$
$$= \frac{\dfrac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \dfrac{2\tan x}{1-\tan^2 x}\tan x}$$
$$= \frac{2\tan x + \tan x(1-\tan^2 x)}{1-\tan^2 x - 2\tan x \tan x}$$
$$= \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

$$\tan 2x = \frac{\sin 2x}{\cos 2x}$$
$$= \frac{2\sin x \cos x}{\cos^2 x - \sin^2 x}$$
$$= \frac{2\tan x}{1 - \tan^2 x}$$

93. $$\frac{\sin x + \sin y}{\cos x + \cos y} = \tan\left(\frac{x+y}{2}\right)$$

LHS: $$\frac{\sin x + \sin y}{\cos x + \cos y}$$

$$= \frac{\cancel{2}\sin\left(\frac{x+y}{2}\right)\cancel{\cos\left(\frac{x-y}{2}\right)}}{\cancel{2}\cos\left(\frac{x+y}{2}\right)\cancel{\cos\left(\frac{x-y}{2}\right)}}$$

$$= \tan\left(\frac{x+y}{2}\right)$$

109.

$$\cos 4x = \cos^2 2x - \sin^2 2x$$

$$= \left((\cos^2 x - \sin^2 x)^2\right) - (2\sin x\cos x)^2$$

$$= \cos^4 x - 2\sin^2 x\cos^2 x + \sin^4 x$$

$$- 4\sin^2 x\cos^2 x$$

$\sin^2 x$

$= 1 - \cos^2 x$

$$= \cos^4 x - 2\cos^2 x + 2\cos^3 x$$

$$+ 1 - 2\cos^2 x + \cos^4 x$$

$$- 4\cos^2 x - 4\cos^3 x$$

$2\sin^2 x\cos^2 x$

$= 2\cos^2 x - 2\cos^3 x$

$$= 2\cos^4 x - 2\cos^3 x$$

$$- 8\cos^2 x + 1$$

$4\sin^2 x\cos^2 x$

$= 4\cos^2 x - 4\cos^3 x$

$$= 2t^4 - 2t^3 - 8t^2 + 1$$

7.4

5. $\sin\theta = \frac{\sqrt{3}}{2}$

$$\theta = \sin^{-1}\frac{\sqrt{3}}{2}$$

$$= \frac{\pi}{3}$$

$$\theta = \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

$$\theta = \frac{\pi}{3} + 2k\pi,\ \frac{2\pi}{3} + 2k\pi$$

17. $\cos\theta = -\frac{\sqrt{3}}{2}$

$\hat{\theta} = \cos^{-1}\frac{\sqrt{3}}{2}$

$= \frac{\pi}{6}$

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\theta = \frac{5\pi}{6} + 2k\pi, \frac{7\pi}{6} + 2k\pi$

$k = 0, \quad \theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

$k = 1, \quad \theta = \frac{17\pi}{6}, \frac{19}{6}\pi$

$k = 2, \quad \theta = \frac{29\pi}{6}, \frac{31\pi}{6}$

21. $\cos\theta = 0.28$

$\theta = \cos^{-1} 0.28$

$= 1.287 + 2k\pi$

$\theta = 2\pi - 1.287$

$= 4.996 + 2k\pi$

$k = -1, \quad \theta = -4.996, -1.287$

$k = 0, \quad \theta = 1.287, 4.996$

$k = 1, \quad \theta = 7.570, 11.279$

23. $\tan\theta = -10$

$$\hat{\theta} = \tan^{-1} 10$$
$$= 1.471$$

$$\theta = -\hat{\theta}$$
$$= -1.471 + k\pi$$

$k=-2$, $\theta = -7.75$

$k=-1$, $\theta = -4.61$

$k=0$, $\theta = -1.47$

$k=1$, $\theta = 1.67$

$k=2$, $\theta = 4.81$

$k=3$ $\theta = 7.95$

27. $\sqrt{2}\sin\theta + 1 = 0$

$$\sqrt{2}\sin\theta = -1$$

$$\sin\theta = -\frac{1}{\sqrt{2}}$$

$$\hat{\theta} = \sin^{-1}\frac{1}{\sqrt{2}}$$

$$= \frac{\pi}{4}$$

$$\theta = -\hat{\theta}$$

$$= -\frac{\pi}{4}$$

$$\theta = \pi - \left(-\frac{\pi}{4}\right)$$

$$= \frac{5\pi}{4}$$

$$\therefore \theta = -\frac{\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi$$

33. $2\cos^2\theta - 1 = 0$

$2\cos^2\theta = 1$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$

for $+$, $\theta = \frac{\pi}{4}$, $2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

$-$, $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$, $2\pi - \frac{3\pi}{4} = \frac{5\pi}{4}$

$\therefore \theta = \frac{\pi}{4} + 2k\pi, \frac{7\pi}{4} + 2k\pi, \frac{3\pi}{4} + 2k\pi,$

$\frac{5\pi}{4} + 2k\pi$

[illegible] $\theta = \frac{\pi}{4} + k\pi, \frac{3\pi}{4} + k\pi$

41. $4\cos^2\theta - 4\cos\theta + 1 = 0$

Let $\cos\theta = x$,

$4x^2 - 4x + 1 = 0$

$(2x-1)(2x-1) = 0$

$x = \frac{1}{2}$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \ 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\therefore \theta = \frac{\pi}{3} + 2k\pi, \ \frac{5\pi}{3} + 2k\pi$

53. $\cos\theta\sin\theta - 2\cos\theta = 0$

$$\cos\theta(\sin\theta - 2) = 0$$

$$\cos^2\theta(\sin\theta - 2)^2 = 0$$

$$\cos^2\theta(\sin^2\theta - 2\sin\theta + 4) = 0$$

$$(1-\sin^2\theta)(\sin^2\theta - 2\sin\theta + 4) = 0$$

Let $\sin\theta = x$,

$$(1-x^2)(x^2-2x+4) = 0$$

$$(x-2)^2(x+1)(x-1) = 0$$

$$x = 2, -1, 1$$

$$\sin\theta = -1, 1, 2$$

$\sin\theta \neq 2$

$$\theta = -\frac{\pi}{2}, \frac{\pi}{2},$$

$$\therefore \theta = -\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi$$

7.5

3. $2\cos^2\theta + \sin\theta = 1$

$2(1-\sin^2\theta) + \sin\theta = 1$

$2 - 2\sin^2\theta + \sin\theta = 1$

Let $x = \sin\theta$,

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$x = -\frac{1}{2}, 1$

$\sin\theta = -\frac{1}{2}, 1$

$\hat{\theta} = \sin^{-1}\frac{1}{2}$

$= \frac{\pi}{6}$

$\theta = -\hat{\theta}$

$= -\frac{\pi}{6}, \pi - \left(-\frac{\pi}{6}\right)$

$= -\frac{\pi}{6}, \frac{7\pi}{6}$

$\theta = \sin^{-1}$

$= \frac{\pi}{2}$

$\therefore \theta = -\frac{\pi}{6} + 2k\pi,$

$\frac{7\pi}{6} + 2k\pi,$

$\frac{\pi}{2} + 2k\pi$

7. $2\sin 2\theta - 3\sin\theta = 0$

$2(2\sin\theta\cos\theta) - 3\sin\theta = 0$

$4\sin\theta\cos\theta - 3\sin\theta = 0$

$\sin\theta(4\cos\theta - 3) = 0$

$\sin\theta = 0$

$\theta = \sin^{-1} 0$

$= 0$

$\theta = 0, \pi$

$\cos\theta = \frac{3}{4}$

$\theta = \cos^{-1}\frac{3}{4}$

$=$

9. $\cos 2\theta = 3\sin\theta - 1$

$$1 - 2\sin^2\theta = 3\sin\theta - 1$$

$$2\sin^2\theta + 3\sin\theta - 2 = 0$$

$$(2\sin\theta - 1)(\sin\theta + 2) = 0$$

$$\sin\theta = \frac{1}{2}, -2 \text{ (undefined)}$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \sin^{-1}\frac{1}{2}$$

$$= \frac{\pi}{6}, \pi - \frac{\pi}{6}$$

$$= \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$$

11. $2\sin^2\theta - \cos\theta = 1$

$2(1-\cos^2\theta) - \cos\theta = 1$

$2 - 2\cos^2\theta - \cos\theta = 1$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{2}, -1$

$\theta = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$

$\therefore \theta = \frac{\pi}{3} + 2k\pi, \pi + 2k\pi,$

$\frac{5\pi}{3} + 2k\pi$

$\hat{\theta} = \cos^{-1} 1$

$= 0$

$\theta = \pi - \hat{\theta}$

$= \pi$

$\theta = \pi + 2k\pi$

13. $\sin\theta - 1 = \cos\theta$

$$(\sin\theta - 1)^2 = \cos^2\theta$$

$$\sin^2\theta - 2\sin\theta + 1 = \cos^2\theta$$

$$\sin^2\theta - 2\sin\theta + 1 - 1 + \sin^2\theta = 0$$

$$2\sin^2\theta - 2\sin\theta = 0$$

$$2\sin\theta(\sin\theta - 1) = 0$$

$\sin\theta = 0$

$\theta = 0, \pi$

$\sin\theta = 1$

$\theta = \sin^{-1} 1$

$= \frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{2} + 2k\pi, \pi + 2k\pi$

17. $2\cos 3\theta = 1$

$$\cos 3\theta = \frac{1}{2}$$

$$\cos(2\theta + \theta) = \frac{1}{2}$$

$$\cos 2\theta \cos\theta - \sin 2\theta \sin\theta = \frac{1}{2}$$

LHS:

$$(2\cos^2\theta - 1)\cos\theta - (2\sin\theta\cos\theta)\sin\theta$$

$$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$$

$$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$$

$$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$$

$$= 4\cos^3\theta - 3\cos\theta$$

$$4\cos^3\theta - 3\cos\theta = \frac{1}{2}$$

$$\cos\theta(4\cos^2\theta - 3) = \frac{1}{2}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$4\cos^2\theta - 3 = \frac{1}{2}$$

$$8\cos^2\theta - 6 = 1$$

$$8\cos^2\theta - 7 = 0$$

$$\cos\theta = \pm\sqrt{\frac{7}{8}}$$

(a) $\theta = \frac{\pi}{6} + k\pi, \frac{5\pi}{6} + k\pi, \frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$

(b) $\theta = \frac{\pi}{6}, \frac{2\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{10\pi}{6}, \frac{11\pi}{6}$

17. $2\cos 3\theta = 1$

$$\cos 3\theta = \frac{1}{2}$$

$$3\theta = \cos^{-1}\frac{1}{2}$$

$$= \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$$

$$= \frac{\pi}{3}, \frac{5\pi}{3}$$

$$= \frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$$

(a) $\theta = \frac{\pi}{9} + \frac{2}{3}k\pi, \frac{5\pi}{9} + \frac{2}{3}k\pi$

(b) $\theta = \frac{\pi}{9}, \frac{7\pi}{9}, \frac{13\pi}{9}, \frac{5\pi}{9}, \frac{11\pi}{9}, \frac{17\pi}{9}$

23. $\cos\frac{\theta}{2} - 1 = 0$

$\cos\frac{\theta}{2} = 1$

$\frac{\theta}{2} = \cos^{-1} 1$

$= 0, 2\pi - 0$

$= 2k\pi$

(a) $\theta = 4k\pi$

(b) $\theta = 0$

35. $f(x) = 3\cos x + 1$,

$g(x) = \cos x - 1$

(b) $f(x) = g(x)$

$3\cos x + 1 = \cos x - 1$

$2\cos x + 2 = 0$

$\cos x + 1 = 0$

$\cos x = -1$

$\hat{x} = \cos^{-1} 1$

$= 0$

$x = \pi + 2k\pi$

$g(\pi) = \cos\pi - 1$

$= -1 - 1$

$= -2$

$\therefore (\pi + 2k\pi, -2)$ for any integer k

Chapter 7 Exercises

1. $\sin\theta\,(\cot\theta + \tan\theta) = \sec\theta$

LHS: $\sin\theta \left(\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta} \right)$

$$= \sin\theta \left(\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} \right)$$

$$= \frac{\cancel{\sin\theta}}{\cancel{\sin\theta}\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta$$

$\therefore$ LHS $=$ RHS